Chiral matrix model for QCD

"Semi" QGP: in QCD, near the chiral phase transition, T: 130 -> 300 MeV

1. Chiral matrix model

2. Suppression of color

3. Dilepton production: *un*suppressed

4. Photon production: *strongly* suppressed

5. Shear viscosity: *strongly* suppressed

The Quark-Gluon Plasma *near* T_c

$T = 0$ to $T \sim 130$ MeV: hadronic resonance model, χ perturbation theory….

$T > 300$ MeV: *resum* perturbation theory

Hard Thermal Loop perturbation theory at *three* loop order

Haque, Bandyopadhyay, Andersen, Mustafa, Mike Strickland, Nan Su, 1402.6907

But: in heavy ion collisions, most time is spent *near* T_c.

Assume Bjorken hydrodynamics: in the central plateau, $T \sim \dfrac{1}{\tau^{1/3}}$

$T_f = 160$ MeV. RHIC, $T_i = 400$ MeV. LHC, $T_i = 600$ MeV.

In Bjorken hydro, as $T_i \to \infty$, $\langle T \rangle \to \dfrac{3}{2} T_f$ = 215 @ RHIC; = 227 @ LHC

Chiral matrix model for QCD

Chiral symmetry

For 3 flavors of massless quarks coupled to a gauge field,

$$\mathcal{L}^{qk} = \overline{q}\, \not{D}\, q = \overline{q}_L \not{D}\, q_L + \overline{q}_R \not{D}\, q_R \quad , \quad q_{L,R} = \frac{1 \pm \gamma_5}{2}\, q$$

Classically, global flavor symmetry of $SU(3)_L$ x $SU(3)_R$ x $U(1)_A$,

$$q_L \to \mathrm{e}^{-i\alpha/2}\, U_L\, q_L \quad , \quad q_R \to \mathrm{e}^{+i\alpha/2}\, U_R\, q_R$$

Simplest order parameter for χ symmetry breaking (χSB'g):
a,b… = flavor. A,B… = color

$$\Phi^{ab} = \overline{q}_L^{\,bA}\; q_R^{aA}$$

$$\Phi \to \mathrm{e}^{+i\alpha}\, U_R\, \Phi\, U_L^\dagger$$

Quantum mechanically, axial $U(1)_A$ is broken by instantons +…. to $Z(3)_A$ at T=0
't Hooft instanton vertex is invariant under $Z(3)_A$:

$$\det \Phi \to \mathrm{e}^{3i\alpha}\, \det \Phi$$

As T → ∞, $U(1)_A$ approximately restored as $1/T^{7 \to 9}$.

Effective Lagrangians for chiral symmetry

Standard linear sigma model for Φ:

$$\mathcal{V}_\Phi = m^2\,\mathrm{tr}\left(\Phi^\dagger\Phi\right) - c_A\left(\det\Phi + \mathrm{c.c.}\right) + \lambda\,\mathrm{tr}\left(\Phi^\dagger\Phi\right)^2$$

Drop $(\mathrm{tr}\ \Phi^+\Phi)^2$. Mass, quartic terms $U(1)_A$ invariant, det Φ under $Z(3)_A$.
For light but massive quarks, need to add

$$\mathcal{V}_H^0 = -\,\mathrm{tr}\left(H\left(\Phi^\dagger + \Phi\right)\right)$$

Quarks generate potential in "q", so *must* couple Φ to quarks: $P_{L,R} = (1 \pm \gamma_5)/2$

$$\mathcal{L}_\Phi^{qk} = \overline{q}\left(\not{D} + \mu\,\gamma^0 + y\left(\Phi\,\mathcal{P}_L + \Phi^\dagger\,\mathcal{P}_R\right)\right)q$$

Use non-perturbative potential from pure glue theory, with *same* $T_d = 270$.
But with quarks, T_d is *just* a parameter in a potential, *not* deconfining T_c.

Similar to Kovacs, Szep, Wolf, 1601.05291; they add vector mesons.

New logarithmic terms

Assume χSB'g occurs, $\langle \Phi \rangle = \phi$, so $m = y\,\phi$.

At $T = 0$, u.v. divergent terms in $4 - \varepsilon$ dim.s:
M = renormalization mass scale

$$\frac{3\,m^4}{16\pi^2}\left(\frac{1}{\epsilon} + \log\left(\frac{M^2}{m^2}\right)\right)$$

Need to add new logarithmic term in Φ:

$$\mathcal{V}_\Phi^{\log} = \kappa \,\mathrm{tr}\,\left[\left(\Phi^\dagger\Phi\right)^2 \log\left(\frac{M^2}{\Phi^\dagger\Phi}\right)\right]$$

To 1 loop order, $\varkappa = 3y^4/(16\,\pi^2)$; we keep it as a free parameter.

In practice, log term complicates the computation, but does not signficantly alter the conclusions from $\varkappa = 0$.

New symmetry breaking term

With just usual symmetry breaking term,
at high T,

$$\mathcal{V}^{\text{eff}} \approx -h\,\phi + \frac{1}{12}\,y^2\,T^2\,\phi^2 + \ldots\;,\; T \to \infty$$

The first is SB'g, the second from fermion fluctuations.
But then there is no symmetry breaking at high T,

$$\phi \sim \frac{12h}{y^2T^2}\;,\; m_{qk} \sim y\phi \sim \frac{1}{T^2}$$

Solve by adding a new temperature dependent term *by hand*

$$\mathcal{V}^{\text{eff}} \approx -h\,\phi - \frac{y}{6}\,m_0\,T^2\phi + \frac{1}{12}\,y^2\,T^2\,\phi^2 + \ldots$$

So $\phi \sim m_0/y$ at high T, $m_{qk} \sim m_0$. In QCD, need to be bit more clever,

$$\mathcal{V}_h^T = -\frac{m_{qk}}{V}\left(\;\mathrm{tr}\;\frac{1}{\not{D} + \mu\,\gamma^0 + y\,\Phi_{ii}}\bigg|_{T\neq 0} - (T=0)\right)\;.$$

Solution at T = 0

Consider first the SU(3) symmetric case, $h_u = h_d = h_s$.
Spectrum. 0^-: singlet η' & octet π. 0^+: singlet σ and octet a_0.
Satisfy a 't Hooft relation:

$$m_{\eta'}^2 - m_\pi^2 = m_{a_0}^2 - m_\sigma^2$$

The anomaly moves η' *up* from the π, but also moves σ *down* from the a_0!

QCD: $\langle \Phi \rangle = (\Sigma_u, \Sigma_u, \Sigma_s)$. From:

$$f_\pi = 93 \; , \; m_\pi = 140 \; , \; m_K = 495 \; , \; m_\eta = 540 \; , \; m_{\eta'} = 960$$

Determine:

$$\Sigma_u = 46 \; , \; \Sigma_s = 76 \; , \; h_u = (97)^3 \; , \; h_s = (305)^3 \; , \; c_A = 4560$$

$$m^2 = (538)^2 - 121\, y^4 \; ; \; \lambda = 18 + 0.04\, y^4$$

Leaves one free parameter, Yukawa coupling "y". Determine from T_χ.

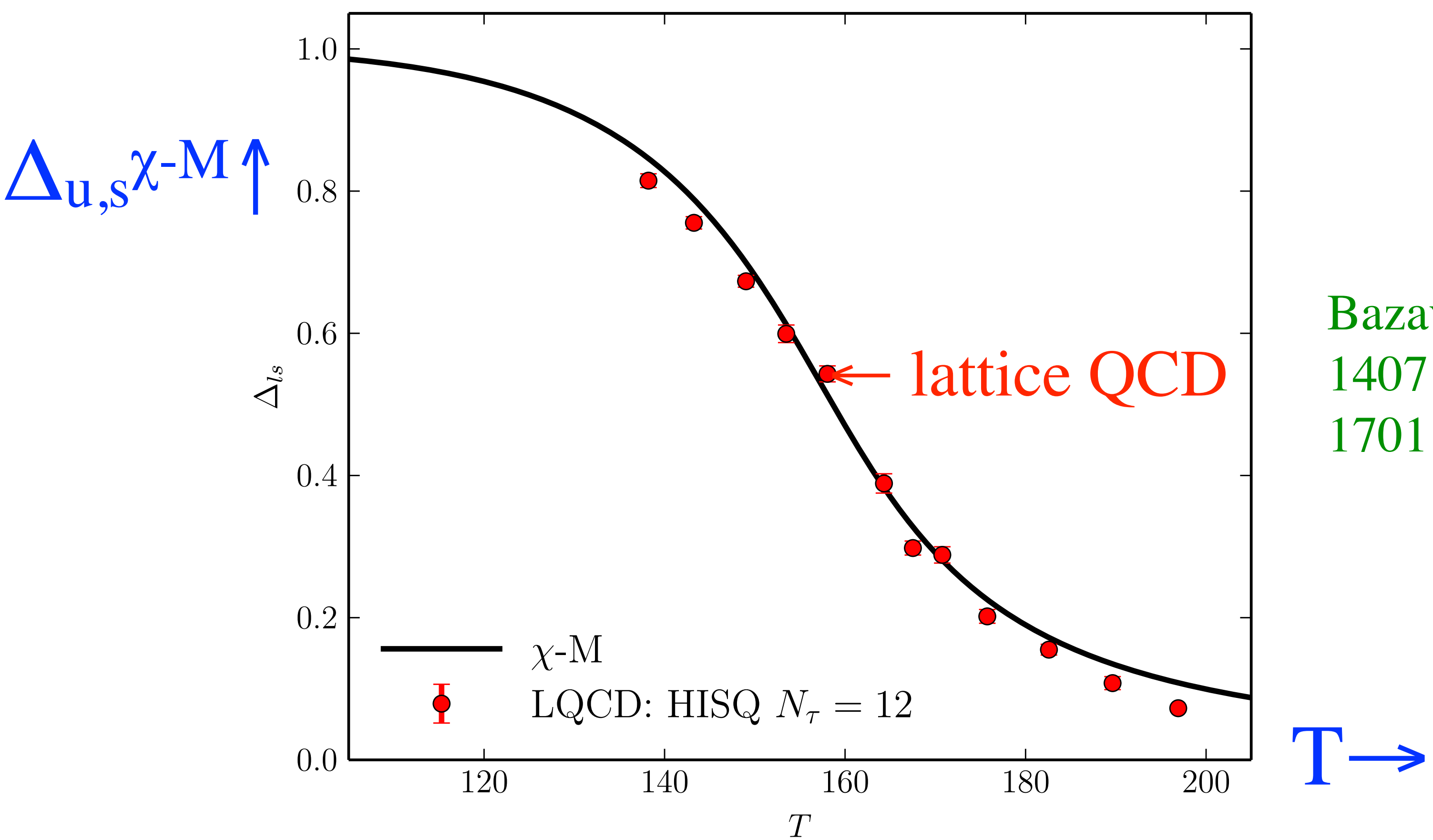
Solution at T ≠ 0

To eliminate u.v. divergences,
lattice uses substracted condensates

$$\Delta_{u,s}^{lattice}(T) = \frac{\langle\overline{q}q\rangle_{u,T} - (m_u/m_s)\langle\overline{q}q\rangle_{s,T}}{\langle\overline{q}q\rangle_{u,0} - (m_u/m_s)\langle\overline{q}q\rangle_{s,0}}$$

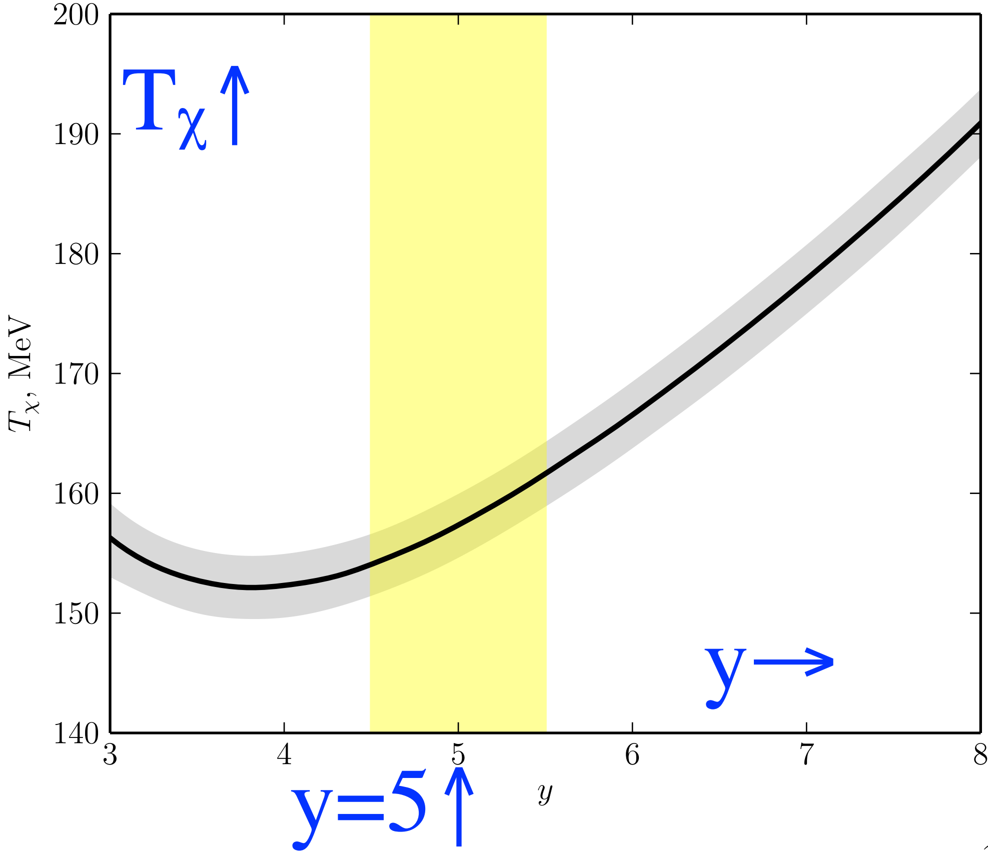
In our model we use analogous quantity
to fix y = 5.

$$\Delta_{u,s}^{\chi-M}(T) = \frac{\Sigma_u(T) - (h_u/h_s)\Sigma_s(T)}{\Sigma_u(0) - (h_u/h_s)\Sigma_s(0)}$$



Bazavov et al,
1407.6387
1701.03548

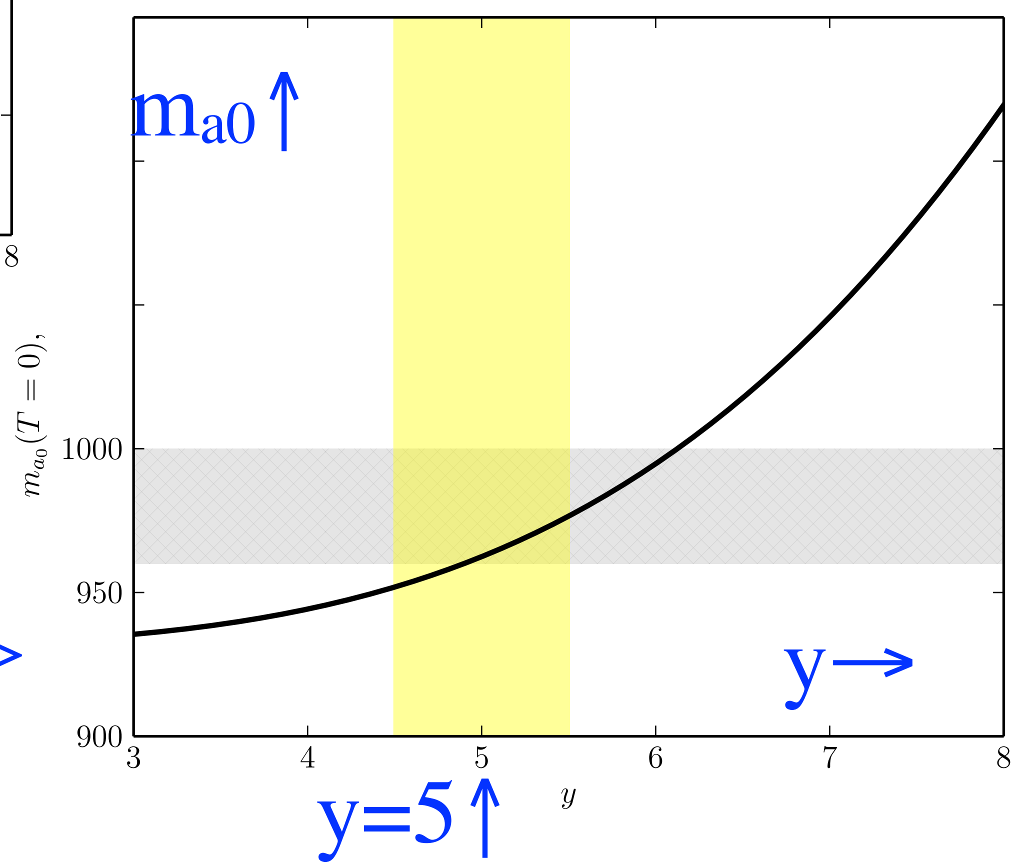
Varying the Yukawa coupling



T_χ defined from maximum in light quark suscep., $d\Sigma_u/dT$

<= Grey band: vary T_d from 260 -> 280

<= Yellow band = y: 4.5 -> 5.5

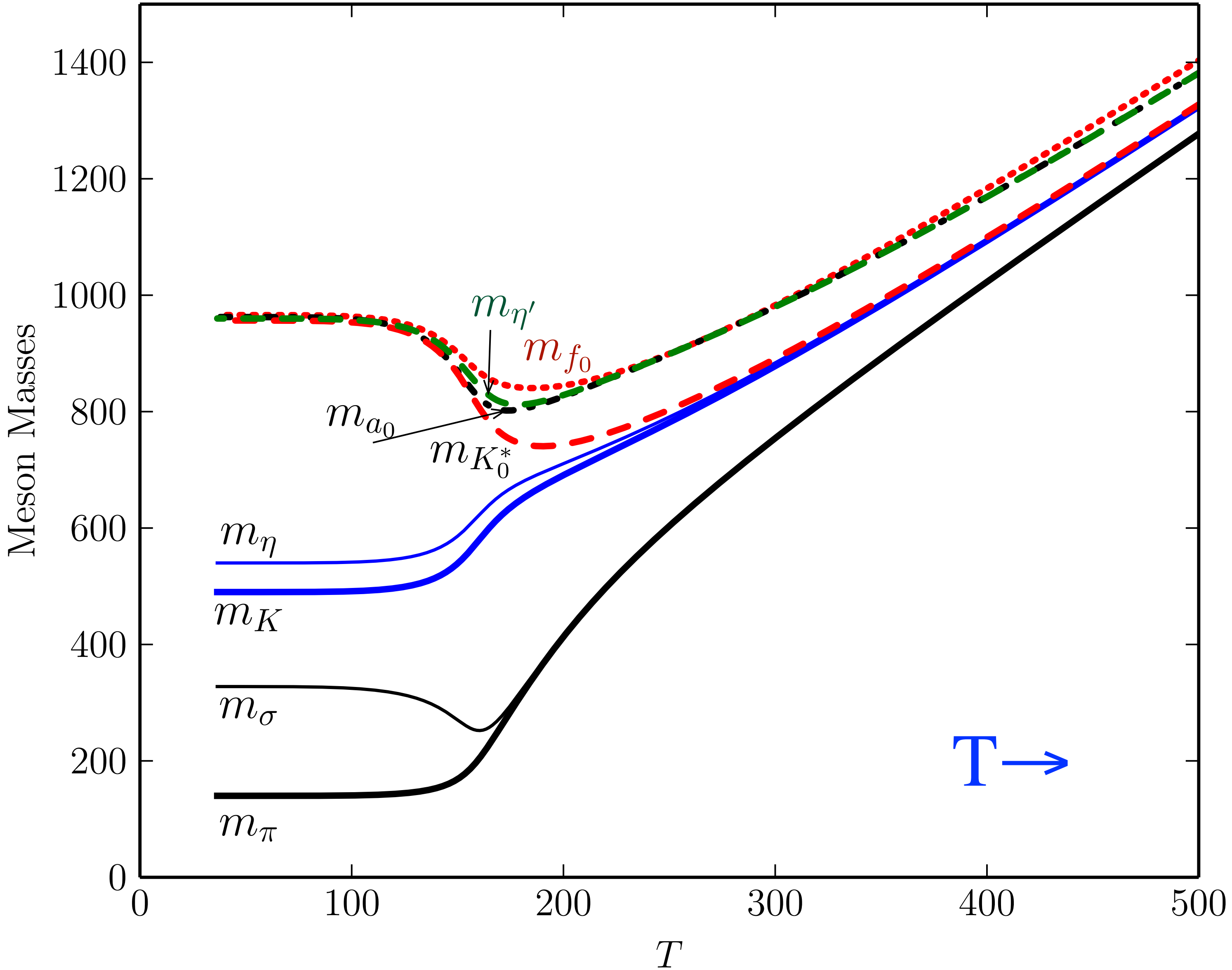


Grey band: experimental uncertainty in the mass of the a_0 =>

Meson masses vs T

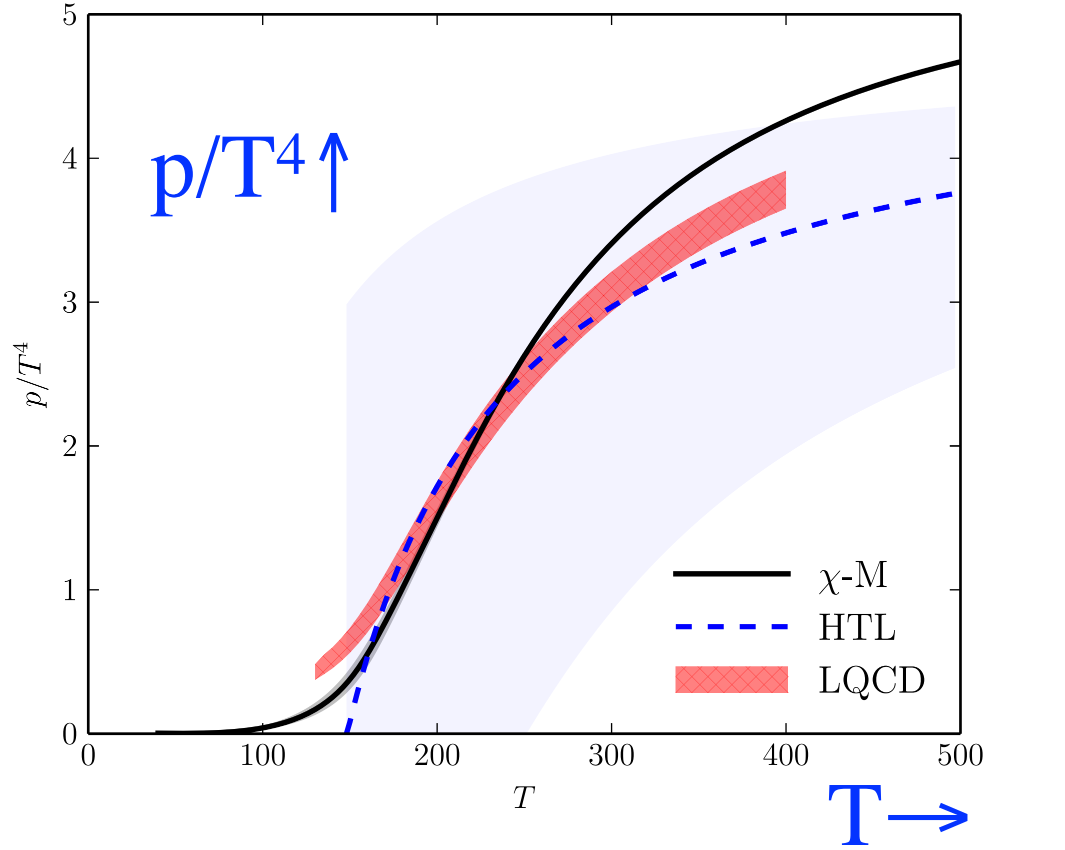
Usual pattern for $m_u = m_d \neq m_s$. y = 5.

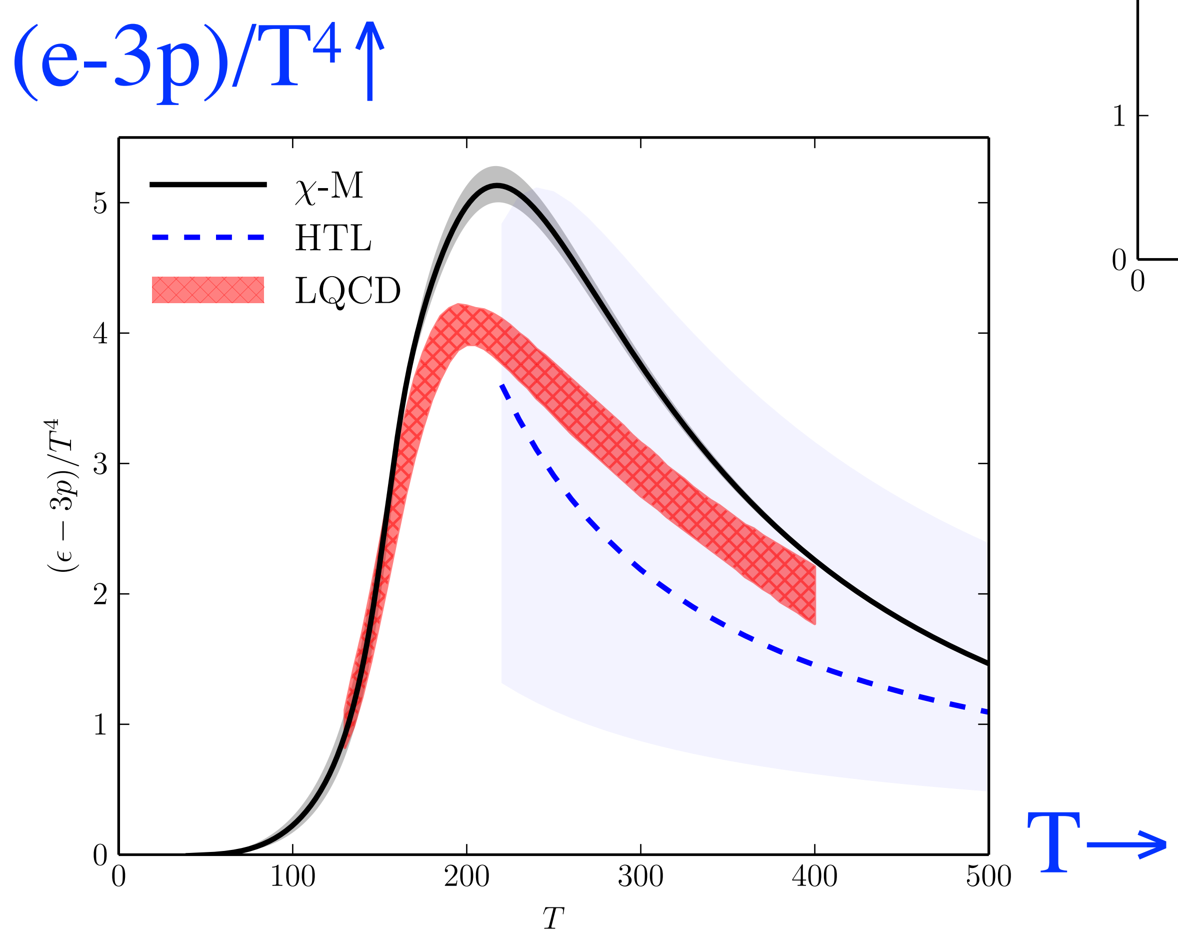
$U(1)_A$ breaking persists to high T, unphysical.



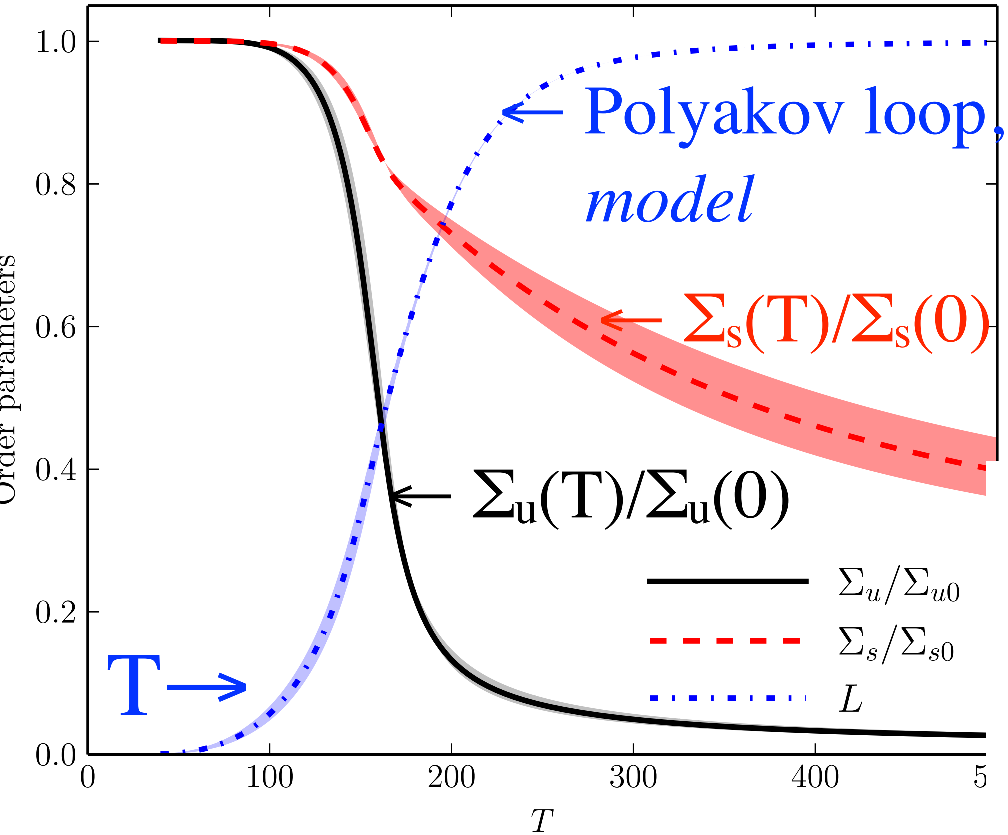
Pressure, interaction measure vs T

Pressure and
interaction measure, $(e-3p)/T^4$,
versus Lattice, Bazavov et al, 1407.6387
and Hard Thermal Loop (HTL)
(blue region = change ren. scale)
Andersen et al, 1511.04660





Order parameters, chiral and deconfining



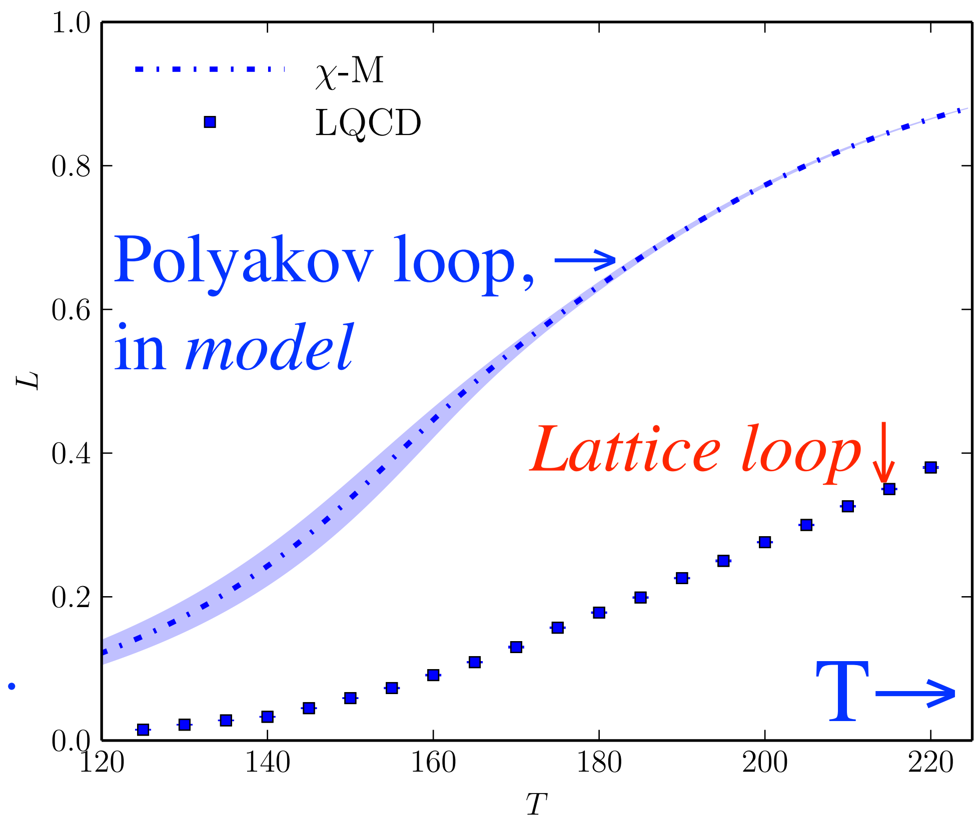
Chiral matrix model:

Chiral and deconfining order parameters are *strongly* correlated

But Polyakov loop from lattice

Petreczky & Schadler, 1509.07874

is *much* smaller than in model.



Persistent discrepancy, as in pure gauge.

To us: what's wrong with lattice loop?

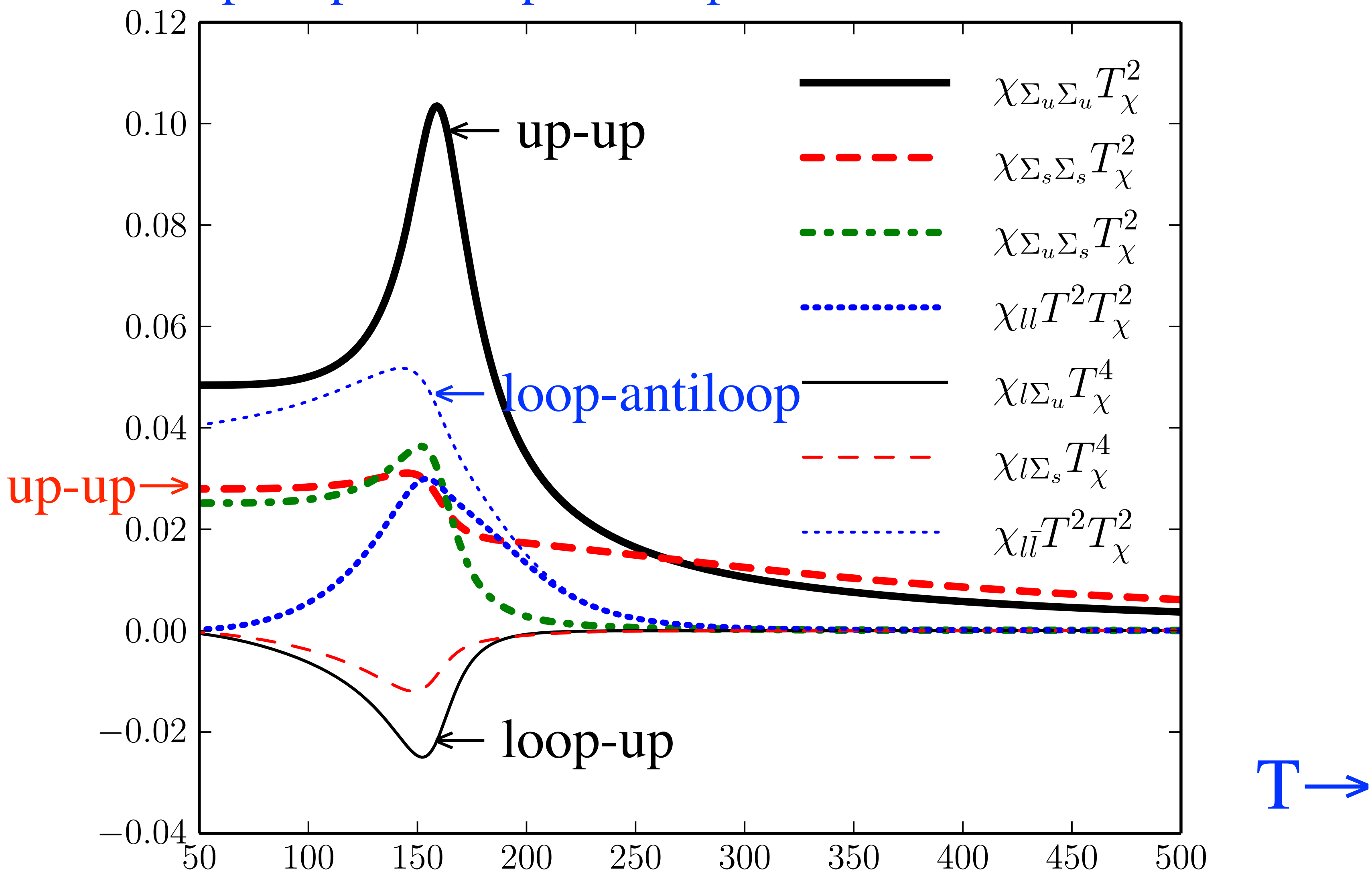
Susceptibilities, chiral and deconfining

Largest peak for up-up; strange-strange small.

In QCD, notable peaks for loop-up & loop-loop, *strongly* correlated with up-up

In chiral limit: loop-up suscep. *diverges*. Sasaki, Friman, Redlich ph/0611147

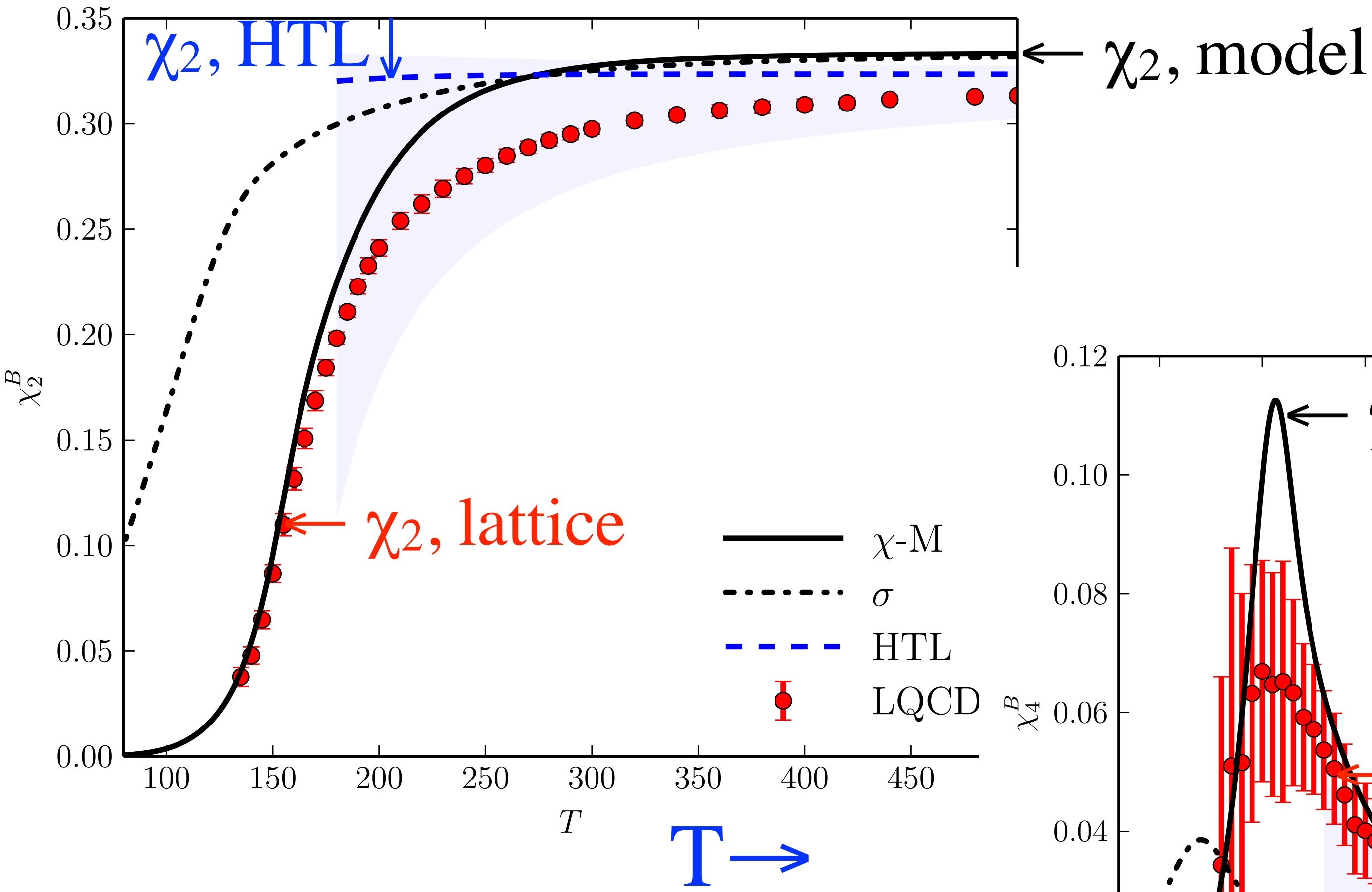
loop-loop and loop-antiloop finite

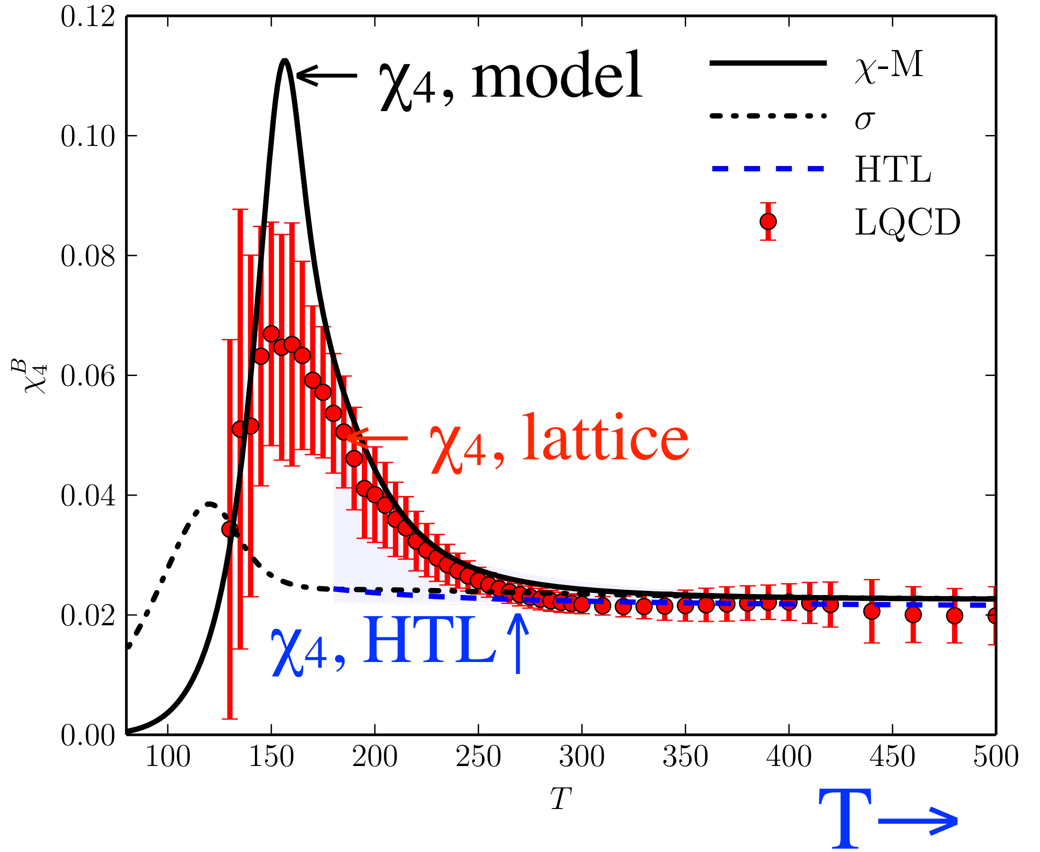


Baryon susceptibilities: 2nd & 4th

As evaluated at $\mu = 0$, lattice ok.
Baryon $\mu_B = 3\ \mu_q$.

$$\chi_n^B(T) = T^{n-4}\ \frac{\partial^n}{\partial \mu_B^n}\ p(T, \mu_B)\Big|_{\mu_B=0}$$

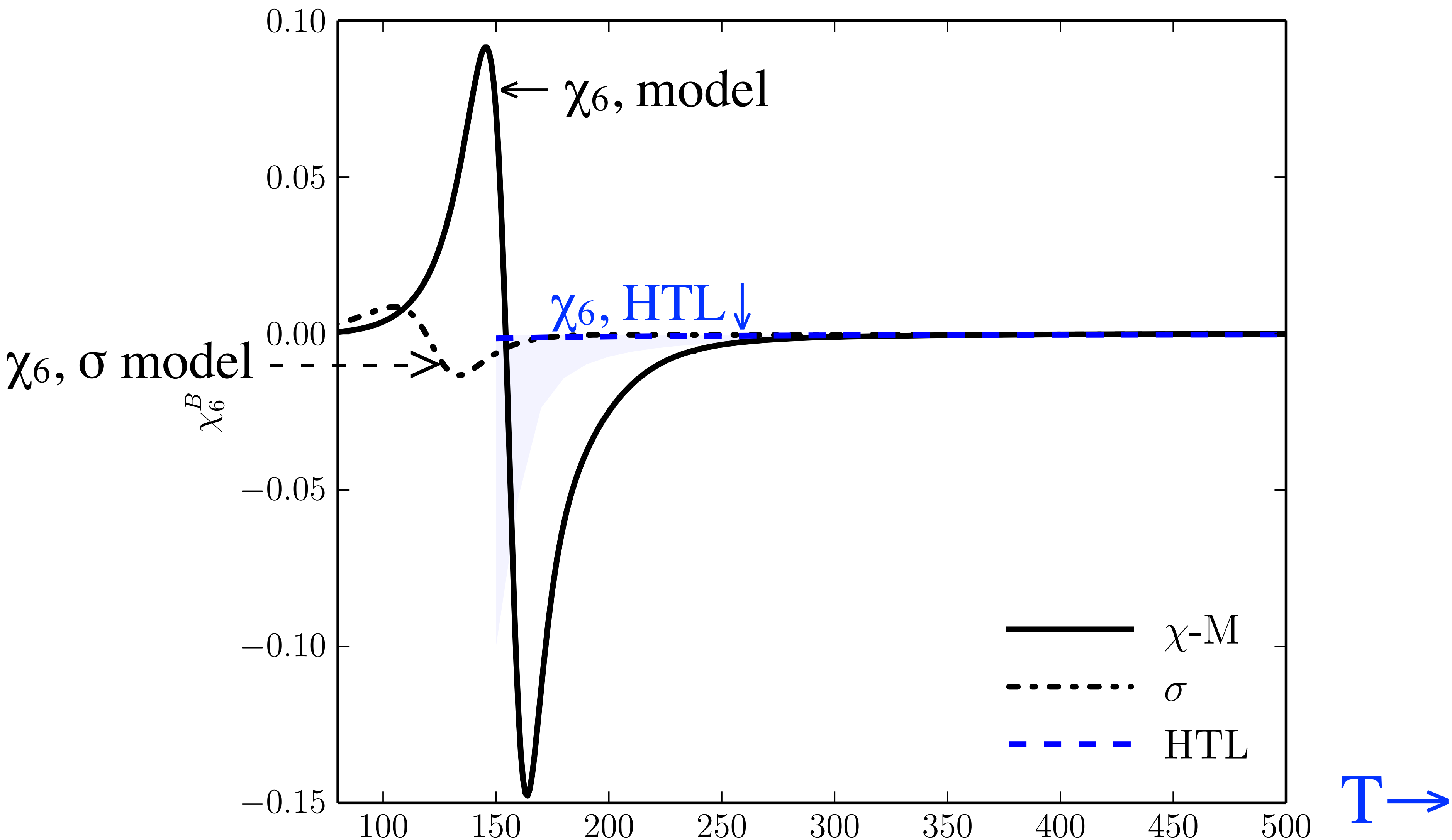




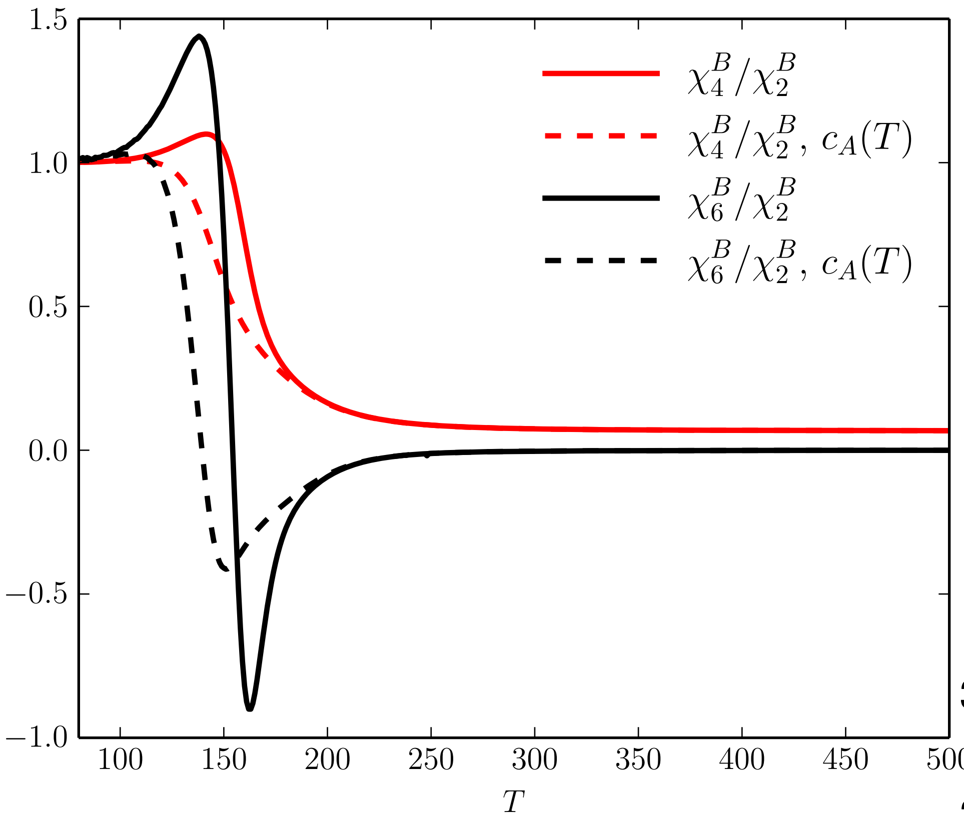
Lattice: Bazavov et al, 1701.04325

6th order baryon susceptibility

In our model, χ_6 shows *non*-monotonic behavior near T_χ.
In HTL, χ_6 is very small (because m=0)
σ model: including change in Σ_u, but *not* in loop. Change in χ_6 *much* smaller.

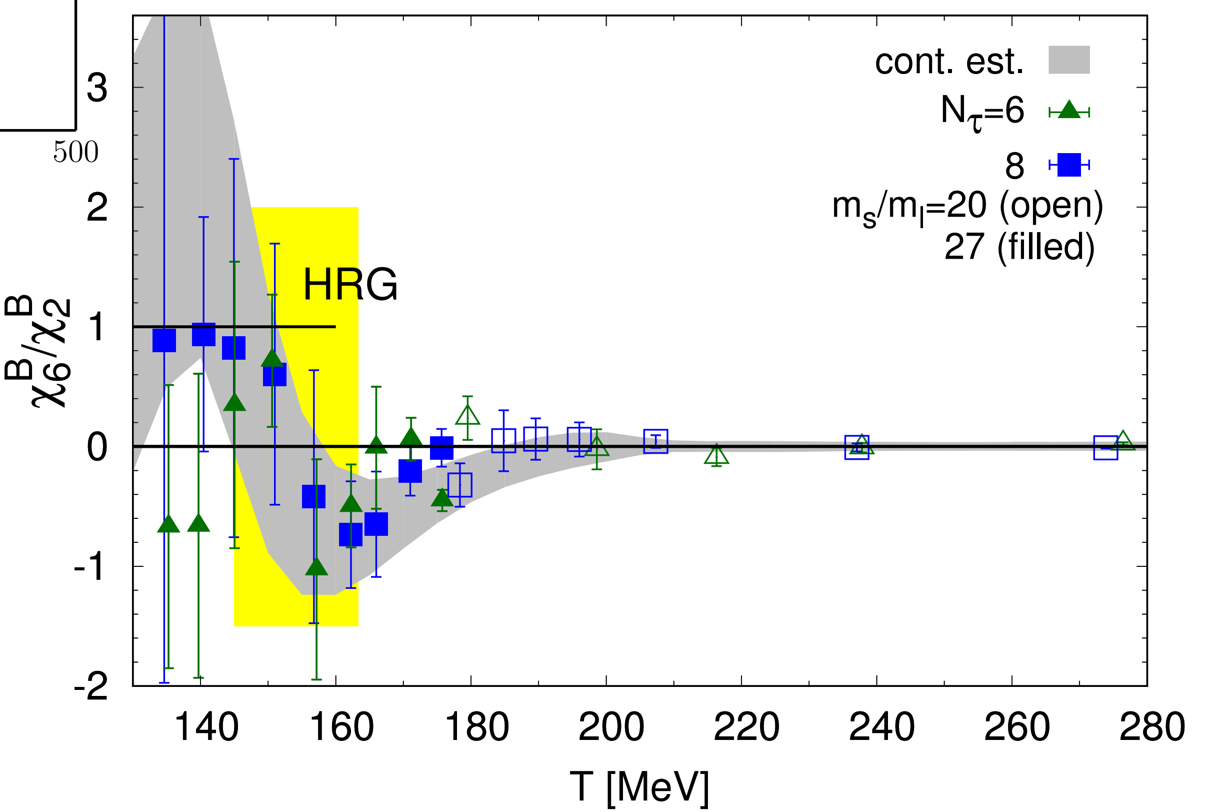


Ratios of moments, vs lattice



Left: ratio of χ_4/χ_2 and $\chi 6/\chi_2$ in model

Bazavov et al, 1701.04325



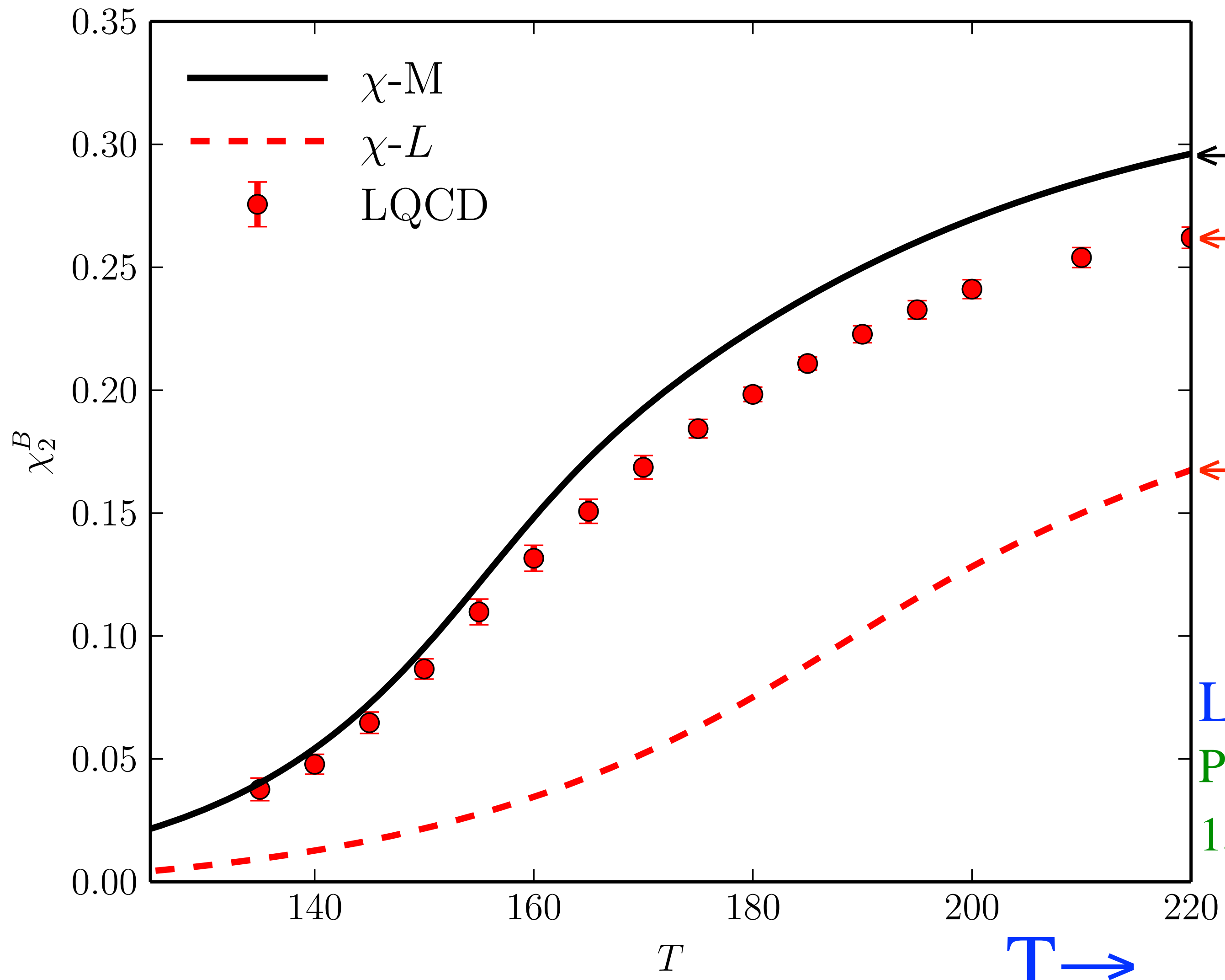
What's up with the lattice loop?

Looked at *wide* variety of possible models.

Below: χ_2 from chiral matrix model, lattice,

and fitting the loop to the lattice value, then computing χ_2.

If the lattice loop is right, then χ_2 is too small.



← χ_2, our model

← χ_2, lattice

← χ_2, *fitting* to lattice loop

Lattice loop:

Petreczky & Schadler, 1509.07874

T→

Suppression of color in the semi-QGP

Suppressing color in the semi-QGP

Statistical distribution functions those for imaginary chemical potential:

$$\widetilde{n}_a(E) = \frac{1}{e^{(E - iQ^a)/T} + 1} \qquad n_{ab}(E) = \frac{1}{e^{(E - i(Q^a - Q^b))/T} - 1}$$

For three colors, color chemical potential:

$$Q^a = \frac{2\pi T}{3}\, q(T)\; (1, -1, 0)$$

When Q ~ T, the *only* soft gluons have $Q^a = Q^b$: *diagonal* elements.

For N colors: ~ N^2 off-diagonal gluons, and ~ N diagonal gluons

In the semi-QGP, soft gluons are suppressed by 1/N.

Suppression of color near T_c

Consider energetic particles, E ≫ T, Boltzmann statistics

$$\widetilde{n}_a(E) \sim e^{-(E-iQ^a)/T} \qquad n_{ab}(E) \sim e^{-(E-i(Q^a-Q^b))/T}$$

While the n(E)'s are complex, sums over color are real.
Polyakov loop:

$$\ell = \frac{1}{N}\sum_{a=1}^{N} e^{iQ^a/T}$$

Summing over color,

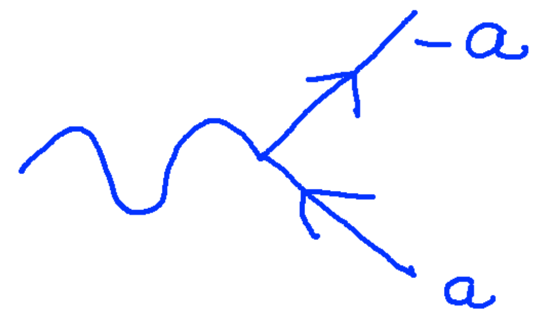
$$\frac{1}{N}\sum_{a=1}^{N}\widetilde{n}_a(E) = e^{-E/T}\,\ell \qquad \frac{1}{N}\sum_{a,b=1}^{N}\widetilde{n}_{ab}(E) = e^{-E/T}\,\ell^2$$

Near T_c, where loop small, quarks suppressed by loop; gluons by loop *squared*.

Dileptons: *un*suppressed

Hard dileptons: same!

Dileptons: off shell photon goes to quark anti-quark pair.
Consider dileptons back to back, total momentum = 0.



Diagrams same, only the distribution functions change.

$$\widetilde{n}_a(E) = \frac{1}{e^{(E - iQ^a)/T} + 1} \qquad \widetilde{n}_{-a}(E) = \frac{1}{e^{(E + iQ^a)/T} + 1}$$

(Imaginary) chemical potential: sign of Q^a *flips* between q and q bar.
Large E: with Boltzmann statistics,

$$\sum_a \widetilde{n}_a(E)\widetilde{n}_{-a}(E) \sim e^{-(E - iQ^a)/T} e^{-(E + iQ^a)/T} = e^{-2E/T}$$

So Q^a's drop out: # dileptons *identical* in deconfined and confined phases!

Soft Dileptons: *more* in confined phase

High T: Q^a=0. As E → 0, # dileptons:
Fermi-Dirac dist. fnc. finite at E = 0.

$$\widetilde{n}(0)^2 \sim \frac{1}{4}$$

In the confined phase, Polyakov loop = 0, find amazing identity:

$$\frac{1}{N}\sum_{a=1}^{N} \widetilde{n}_a(E)\, \widetilde{n}_{-a}(E) \sim n(E) =_{E\to 0} \frac{T}{E}$$

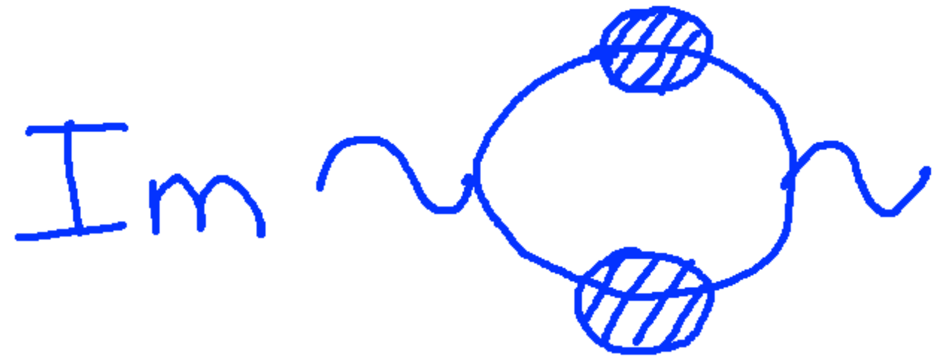
More dileptons in the confined phase!
Confined phase only in the pure gauge theory, but interesting point of principle.

"Statistical confinement": quark anti-quark forms "boson",
which exhibits Bose-Einstein enhancement. But *no* dynamics of confinement.

N.B.: in dynamical quasi-particle model, as T → T_c quarks heavier,
but width increases, so also obtain enhanced dilepton rate.

Dileptons

Explicitly, we computed the diagram:
Here, propagators with hatched dot are
just $p_0 \rightarrow p_0 - i\, Q^a$. *Very* straightforward



$$f_{\ell\bar{\ell}} = \#\ \text{dileptons}\left(\frac{Q \neq 0}{Q = 0}\right)$$

$$f_{\ell\bar{\ell}} = 1 - \frac{2\,T}{3\,p}\log\frac{1 + 3\,\ell\, e^{-p_-/T} + 3\,\ell\, e^{-2p_-/T} + e^{-3p_-/T}}{1 + 3\,\ell\, e^{-p_+/T} + 3\,\ell\, e^{-2p_+/T} + e^{-3p_+/T}}$$

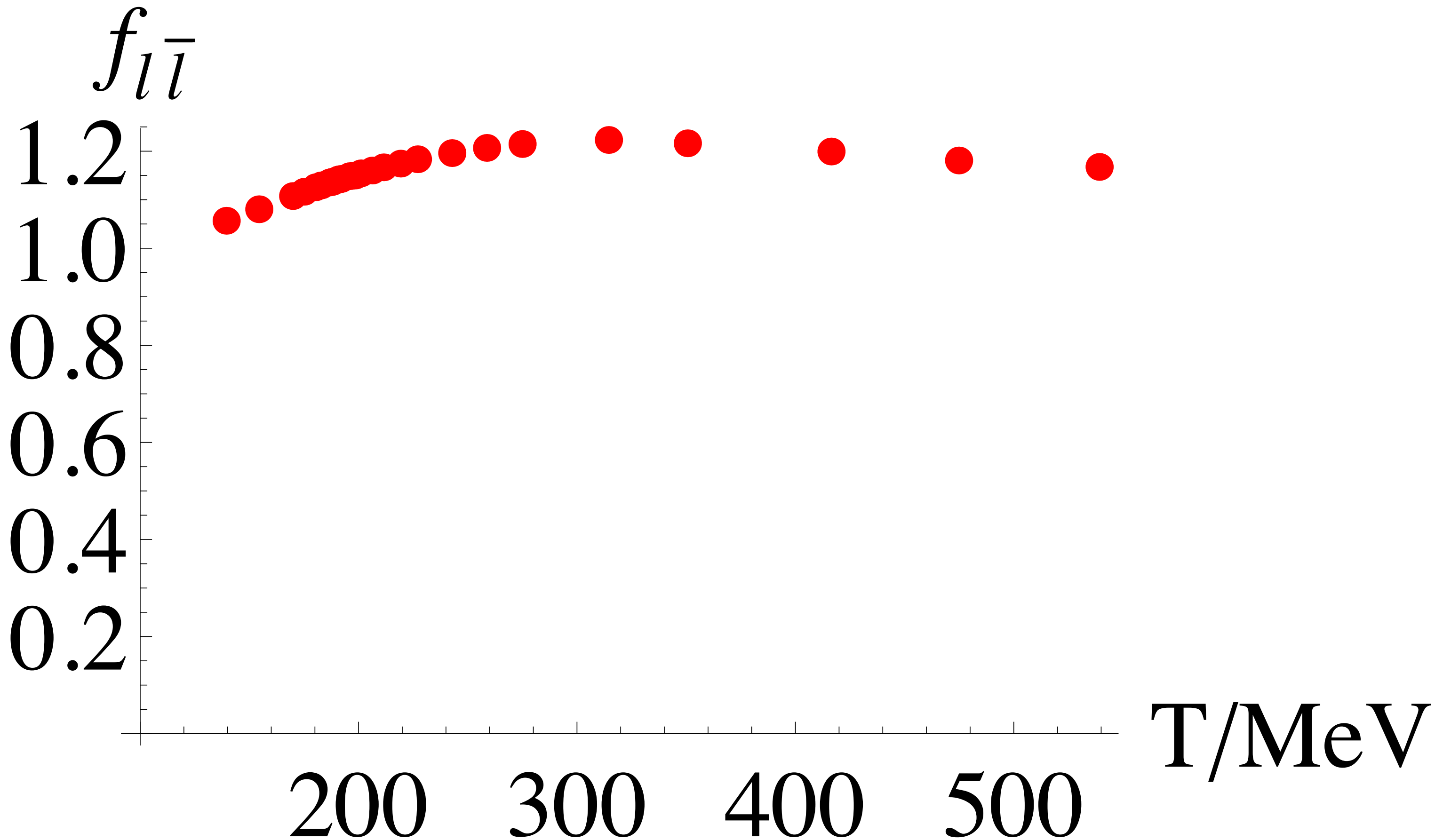
When Q = 0, # dileptons ~ α_{em}. Photon momentum = (E,p), $E_\pm = (E \pm p)/2$.
Polyakov loop = ℓ : = 1 in the perturbative QGP, and = 0 in the confined phase.
Above factor analogous to PNJL model,

Abishek Atreya, Sarkar, Srivastava, 1111.3027, 1404.5697, & Das, 1406.7411

Ratio # dileptons, vs T

Below ratio of # dileptons, vs T. Ratio semi-QGP/perturbative QGP.
Take QCD coupling same, so only function of Q^a's, taken from the lattice.
Mild enhancement of dileptons at small E.

Lee, Wirstam, Zahed, Hansson, ph/9809440:
Condensate in $\sim \langle A_0^2 \rangle$; equivalent to expanding to $\sim \langle Q^2 \rangle$.

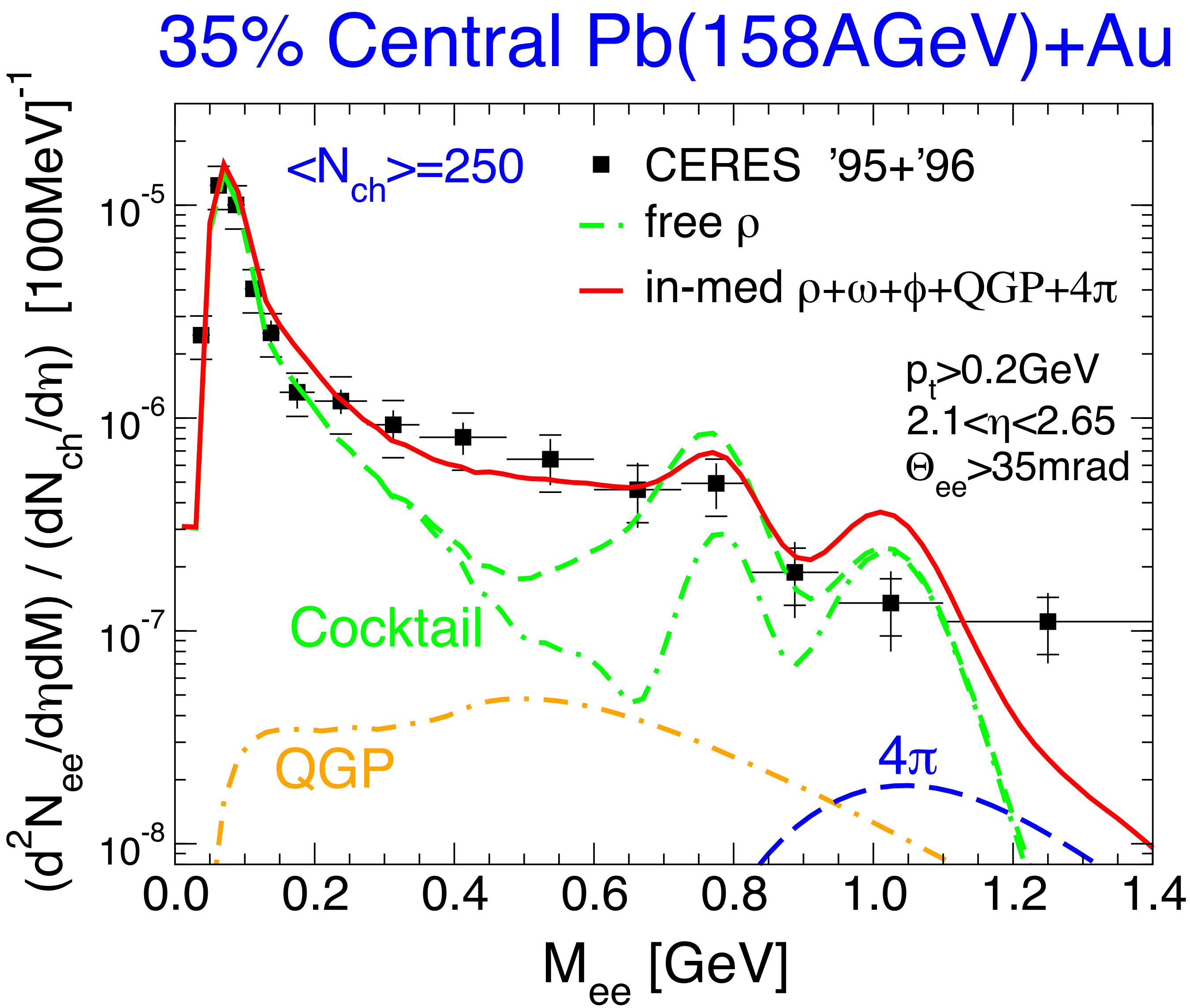


Experiment: dilepton excess below the ϱ

CERES/NA45, $\sqrt{s}$ = 8.8 GeV/A.

Below the ϱ, QGP small, dominated by hadronic cocktail.

Need medium broadened ϱ to fit data: so *need* to fit semi-QGP to hadronic phase



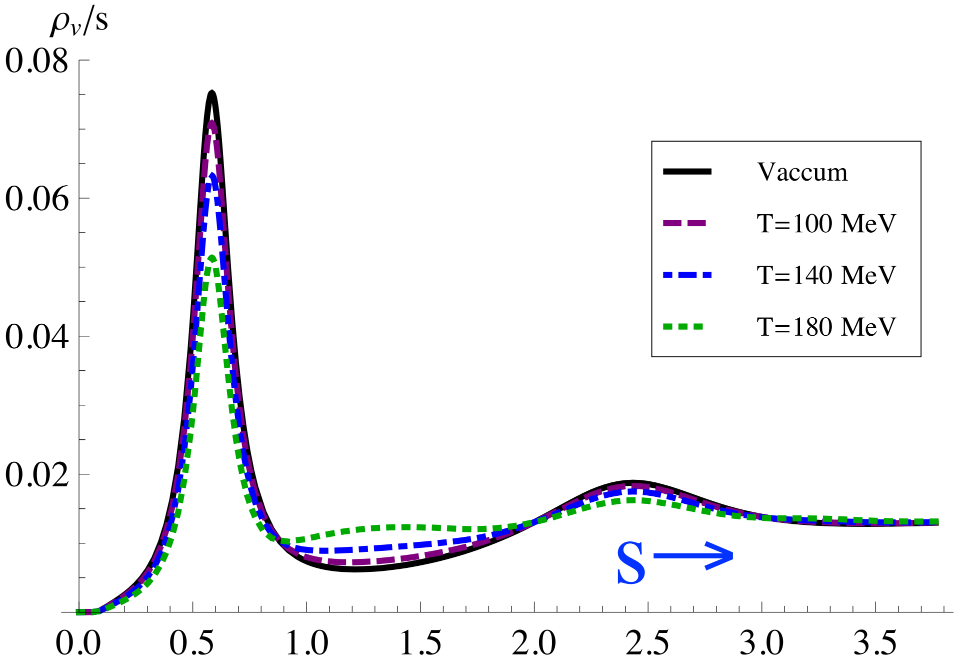
Rapp, 1306.6394

Where does the ϱ go?

As $T \rightarrow T_\chi$, χ symmetry $\Rightarrow$ ϱ and a_1 spectral densities degenerate. *But how?*

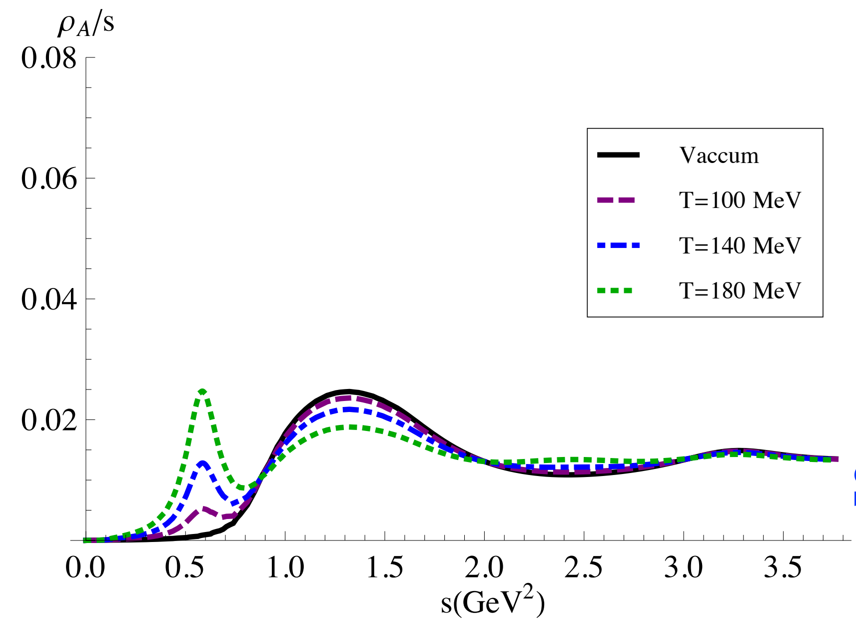
Brown & Rho (PRL'91) ϱ goes *down*. RDP, ph/9503328: ϱ goes *up*.

Holt, Hohler, & Rapp, 1210.7210: ϱ and a_1 peaks *don't* move, just broaden: ?



ϱ spectral density ↑

a_1 spectral density ↓



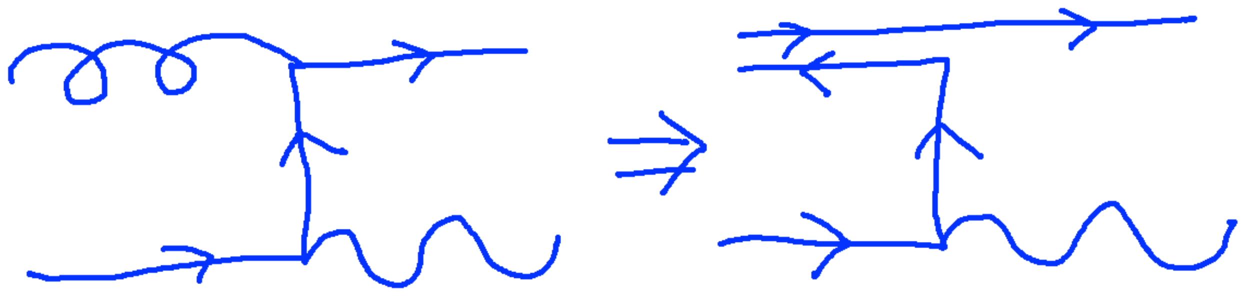
s→

Ayala, Dominguez, Loewe, Mizher, Zhang, 1210.2588, 1309.4135, 1405.2228: find the ϱ *does* move….down :(

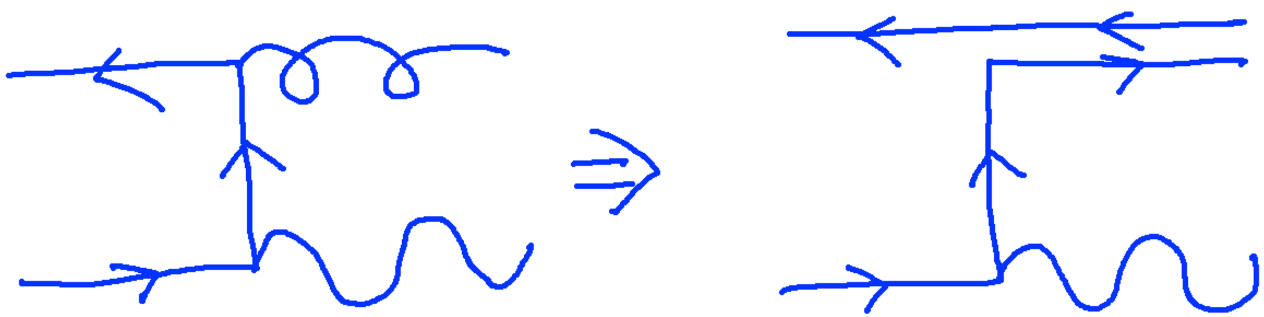
Real photon production: strongly suppressed

Production of hard photons

Photon on the mass shell cannot go to quark anti-quark; must also emit a gluon
At leading order, two processes. Compton scattering:

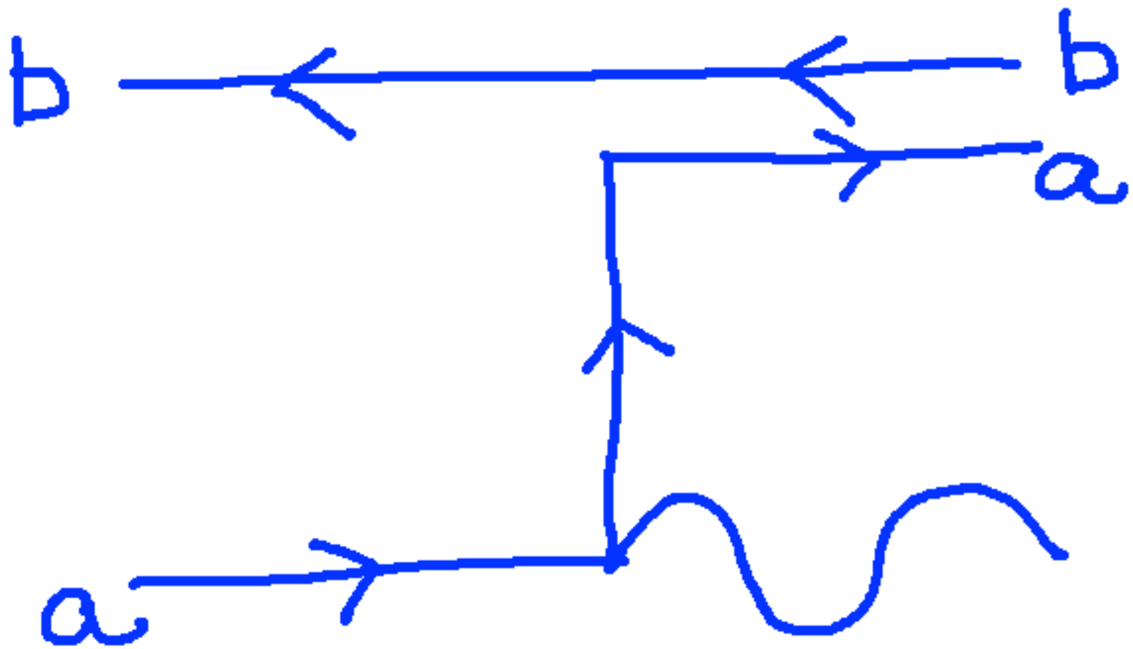


Pair annihilation:



Suppression in confined phase by $1/N^2$

In double line notation: diagram suppressed by loop unless colors of quark and anti-quark the same, a = -b :



But if a = - b, diagonal gluon, suppression of 1/N.

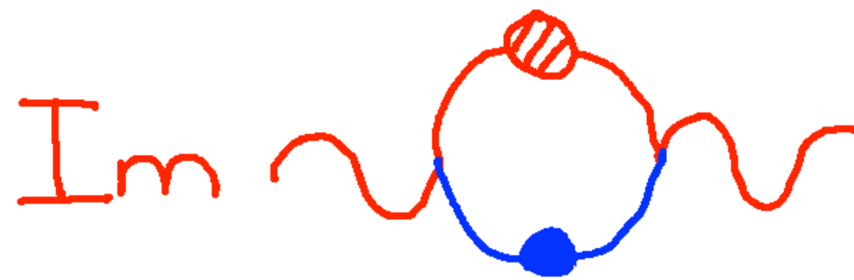
And, if a = -b, tracelessness of gluon implies extra factor of 1/N, or $1/N^2$ in all.

Similar suppression for Compton scattering.

Photon production: computation

Photon momentum “hard”, $P = (E, p)$, $E = p \gg T$. Denote by red lines.
Internal lines can be soft, E or $p \sim T$; denote by blue lines.

Diagrams with one soft quark line:
Hatched blob: $Q^a \neq 0$
Solid blob: HTL with $Q^a \neq 0$
Exhibits logarithmic UV divergence, when
the soft quark line becomes hard.



Also two loops diagrams, in which all lines are hard.
All lines below should be hatched, with $Q^a \neq 0$.
Exhibits logarithmic IR divergence, when the gluon line becomes soft.

Strong suppression of real photons in the confined phase

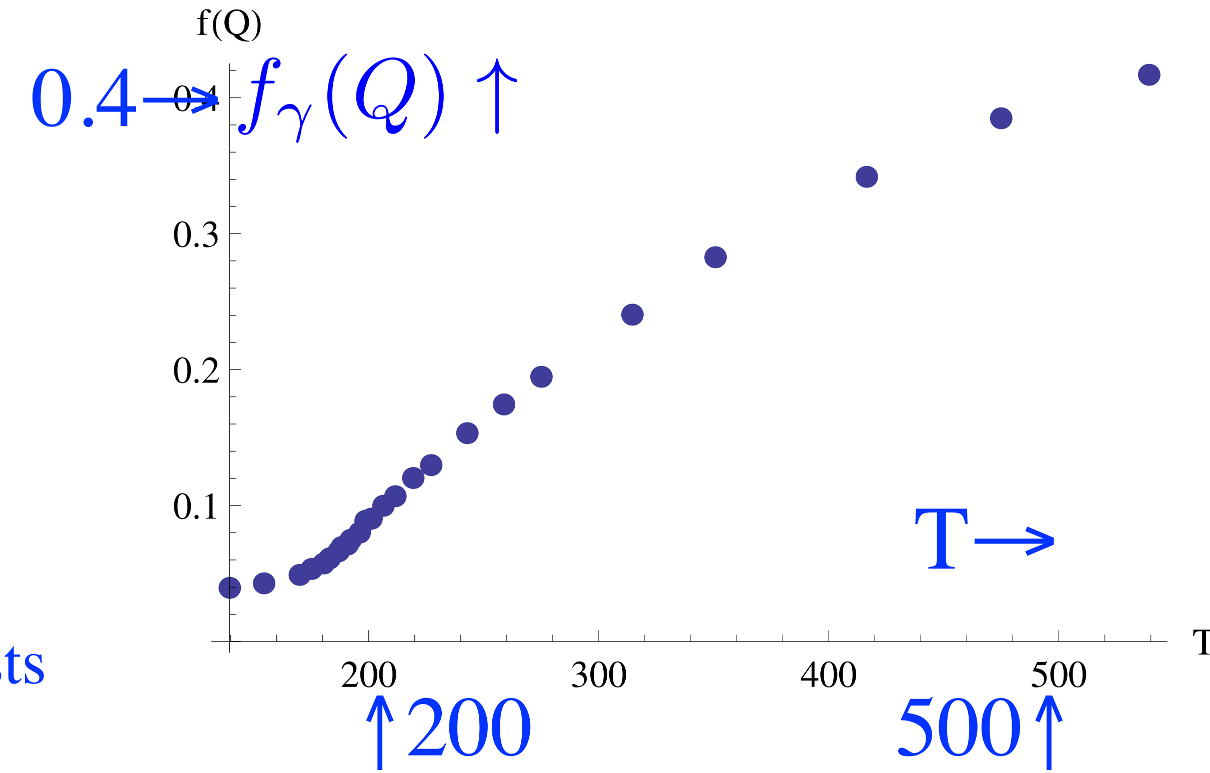
Summing soft + hard, logarithms cancel. For hard photons, very simple result:

$$f_\gamma(Q) = \#\ \text{photons}\left(\frac{Q \neq 0}{Q = 0}\right) = 1 - 4q + \frac{10}{3}q^2\ ;\ q = \frac{Q}{2\pi T}$$

In the confined phase, $q_{\rm conf} = 1/3$, find *huge* suppression:

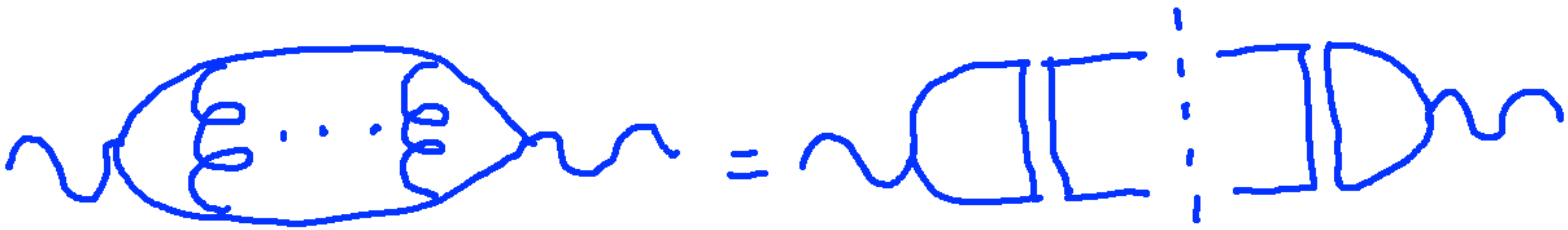
$$f_\gamma(q_{conf}) = \frac{1}{3N^2} = \frac{1}{27}$$

Suppression is so large that it persists even to T ~ 500 MeV.



Landau-Pomeranchuk-Migdal

In the perturbative QGP, even at leading order in g^2, LPM $\Rightarrow$ need to resum an *infinite* set of ladder diagrams: Arnold, Moore & Yaffe, ph/0111107, ph/0204343



Each new rung is down by g^2, but for soft gluon, $k \sim gT$, compensated by Bose-Einstein enhancement times energy denominator,

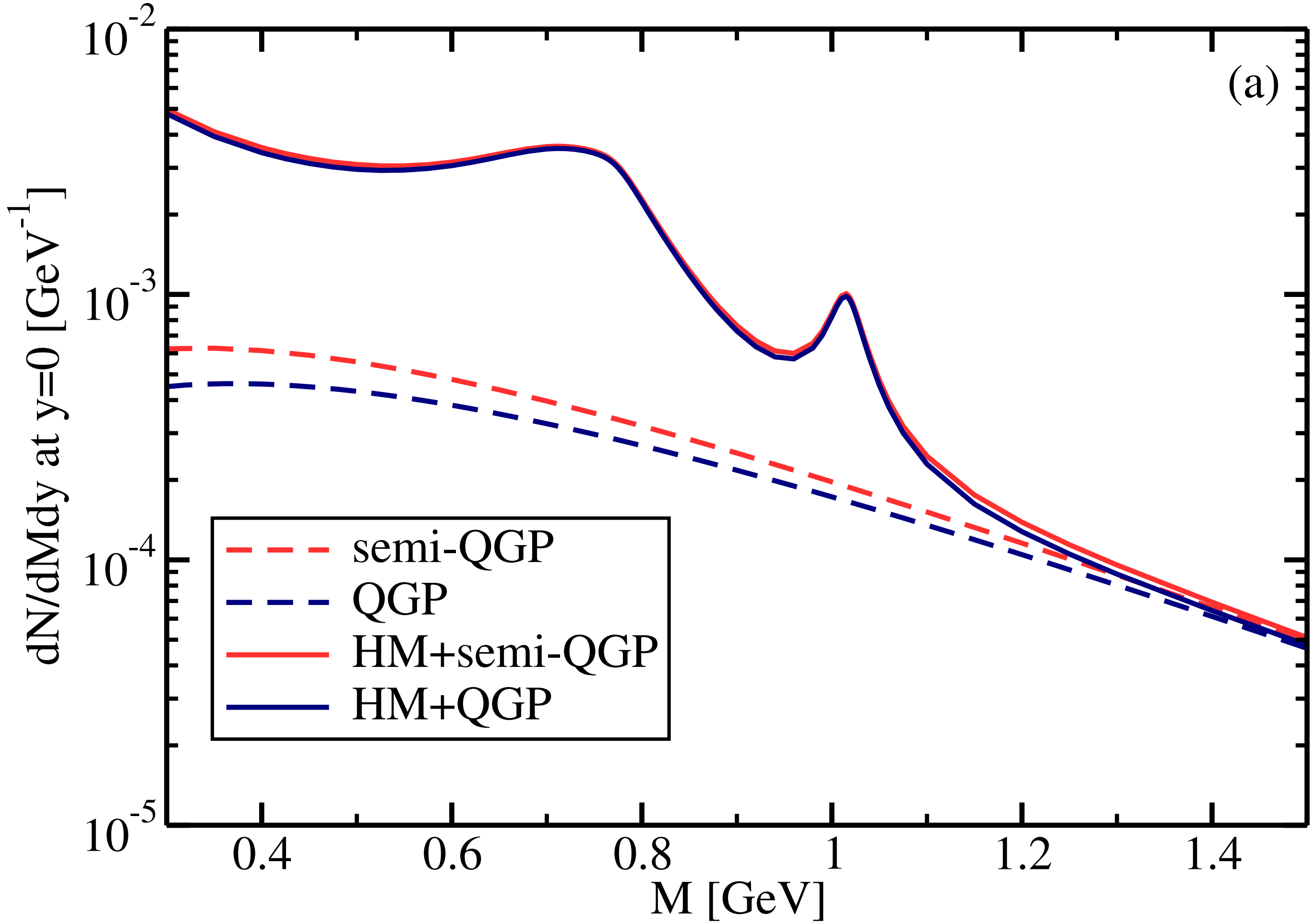
$$g^2\ n(gT)\ \frac{T}{ip_0 - E_k + E_{p-k}} \sim g^2 \frac{T}{gT}\ \frac{T}{gT} \sim 1$$

Semi-QGP: only soft gluons are *diagonal*, so LPM is suppressed by 1/N.
What we did: only $2 \rightarrow 2$ processes, at leading logarithmic order.
Did compute LPM correction, term is large for N = 3.

Hydrodynamics: # dileptons

MUSIC: 3+1 hydro @ RHIC: $\sqrt{s}$ = 200 GeV/A, central collisions
Preliminary analysis: only ideal hydro.

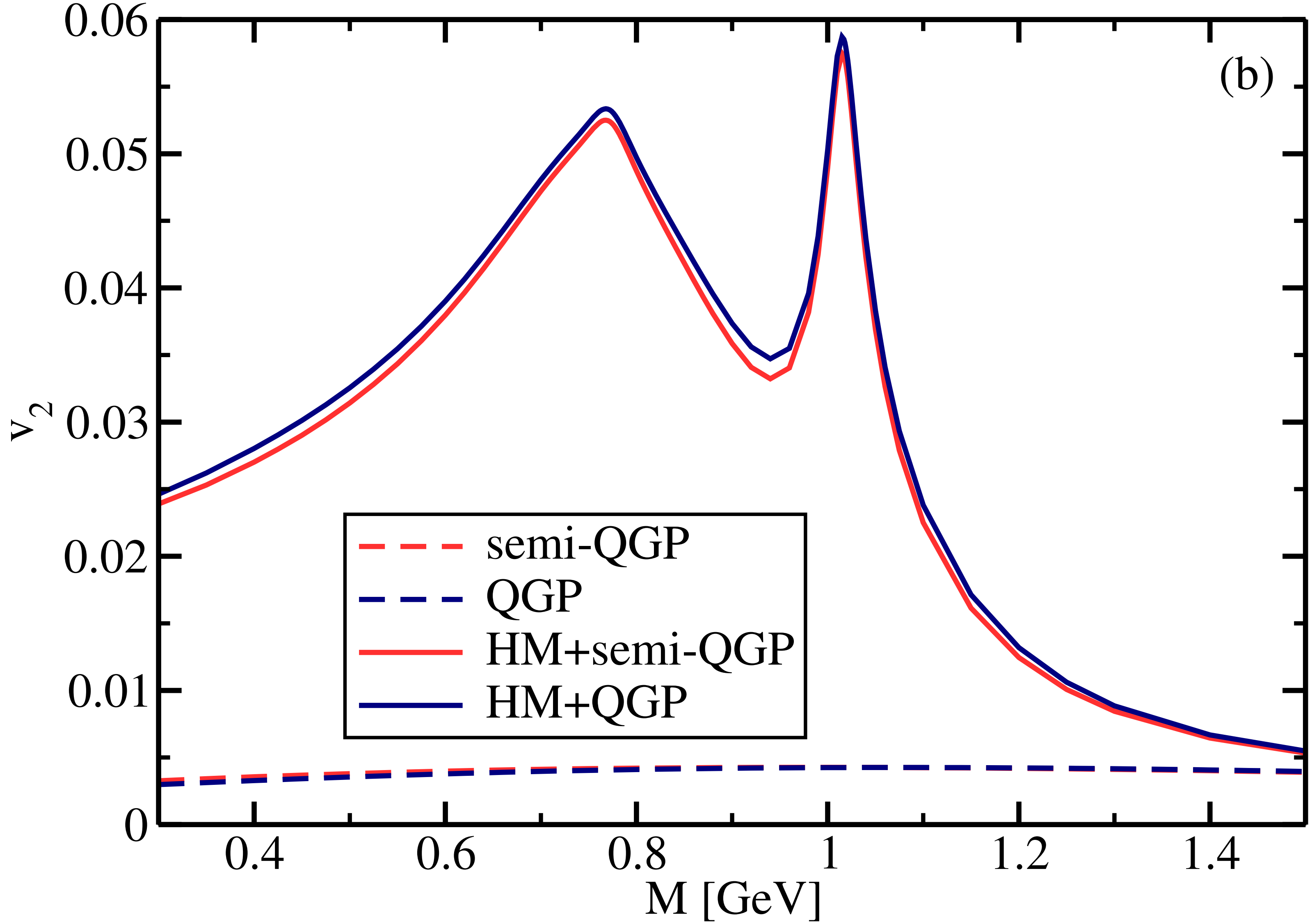
Small enhancement of dileptons in semi-QGP, swamped by hadronic phase.
No matching of semi-QGP to hadronic phase: clearly essential.



Hydrodynamics: dilepton v_2

Since # dileptons dominated by hadrons, effect on elliptic flow, v_2, small.

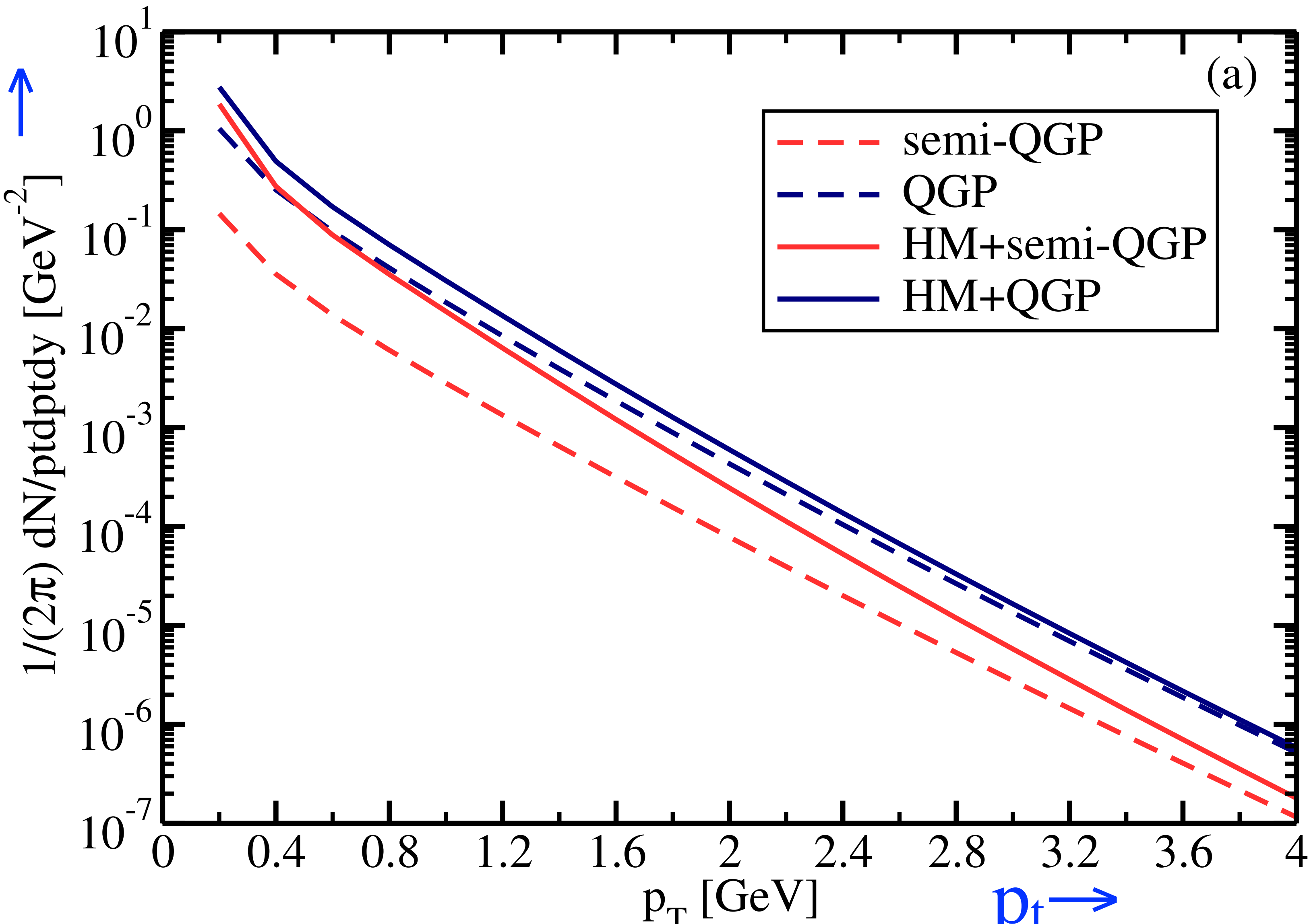
$v_2 \uparrow$



Hydrodynamics: # photons

In semi-QGP, *far* fewer photons above T_c.

photons↑

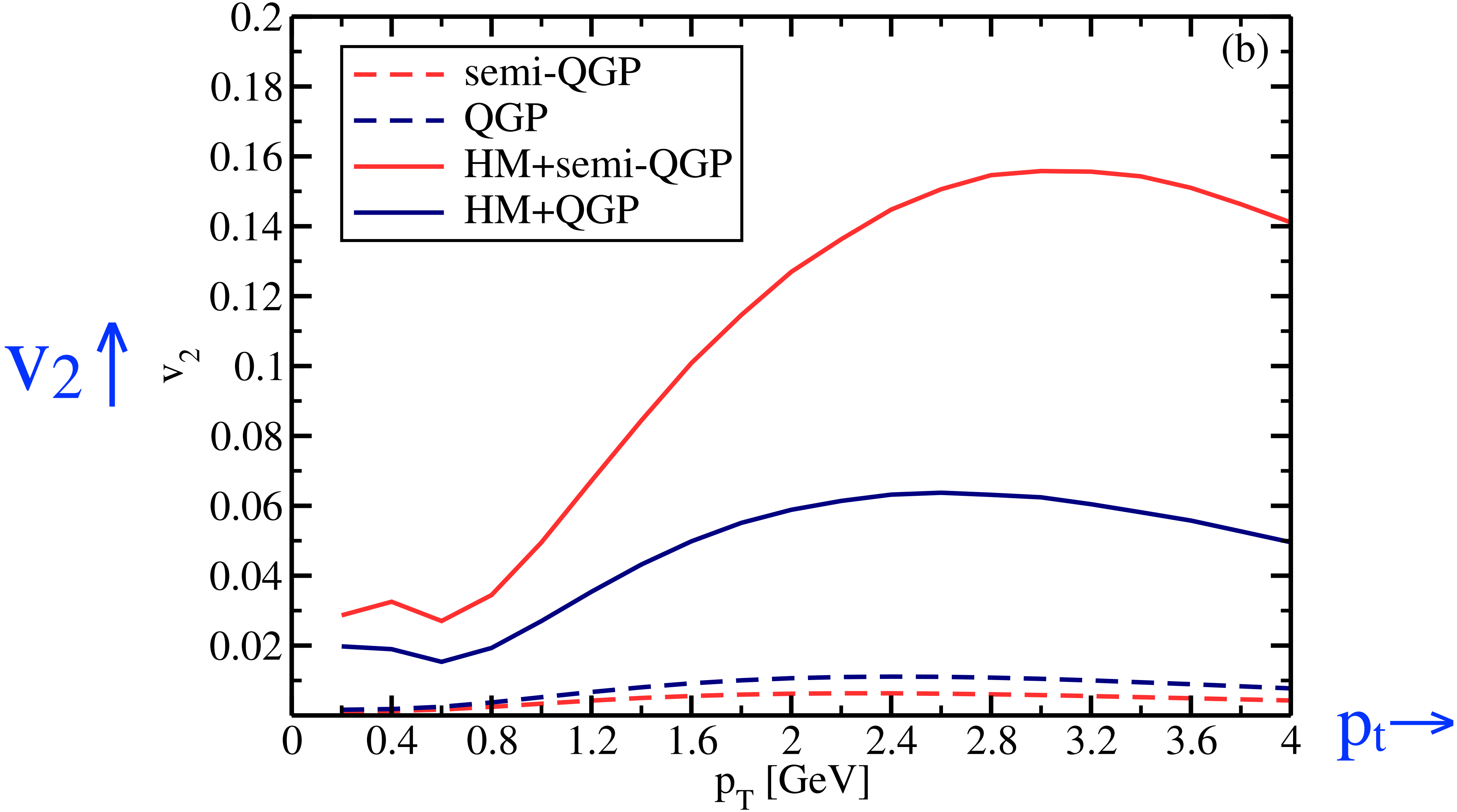


p_t ⟶

Hydrodynamics: photon elliptic flow, v_2

Fewer photons near T_c in semi-QGP has a big effect on the total v_2.

Tends to bias the total v_2 to that in hadronic phase. Small "dilution" by QGP.



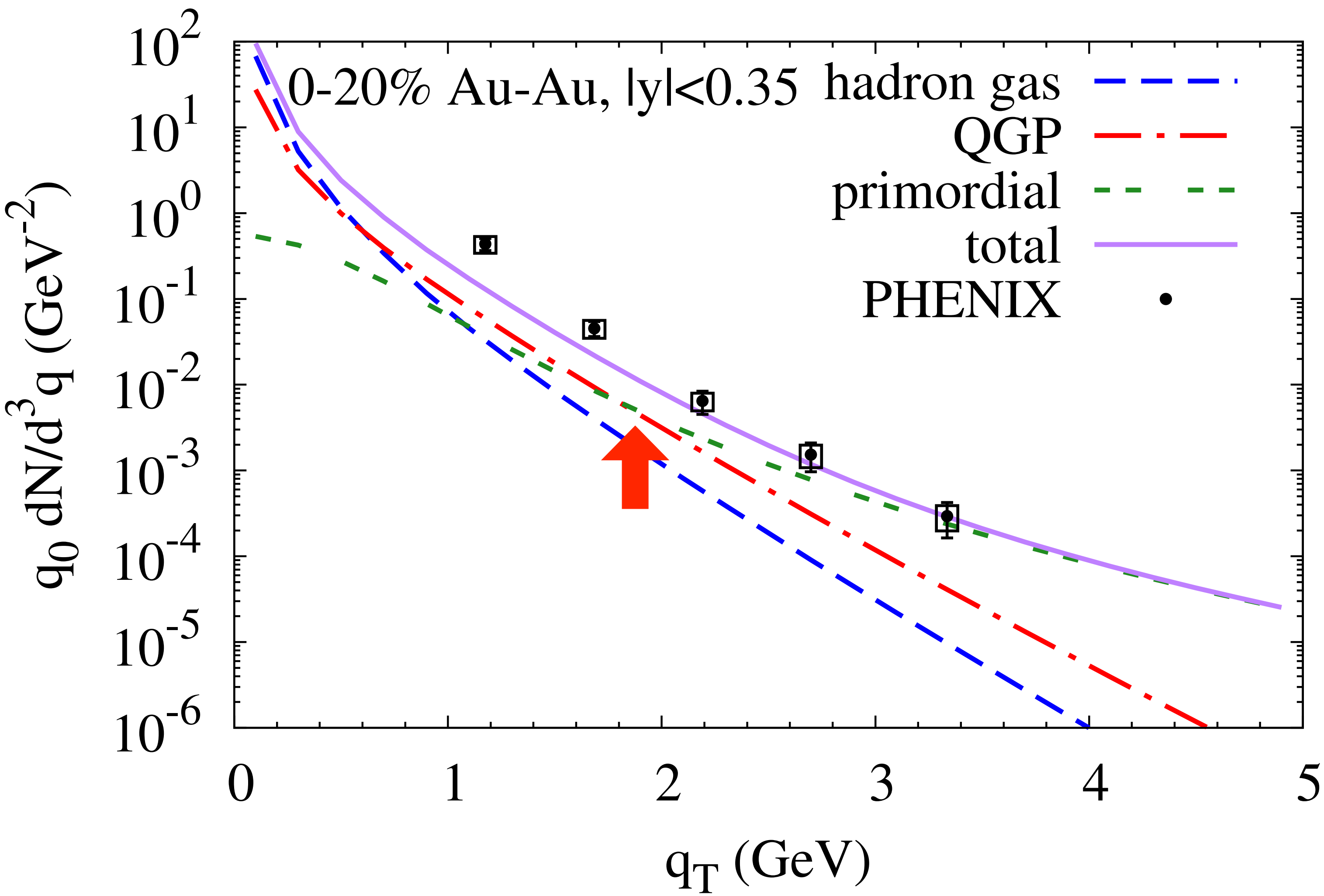
PHENIX vs theory: puzzle of the “missing” photons

Sources of photons: QGP, hadron gas, “primordial” = hard initial processes

PHENIX: more photons than expected?

At RHIC: “primordial” photons appear to dominate above $p_t \sim 2$ GeV

van Hees, He, Rapp, 1404.2846

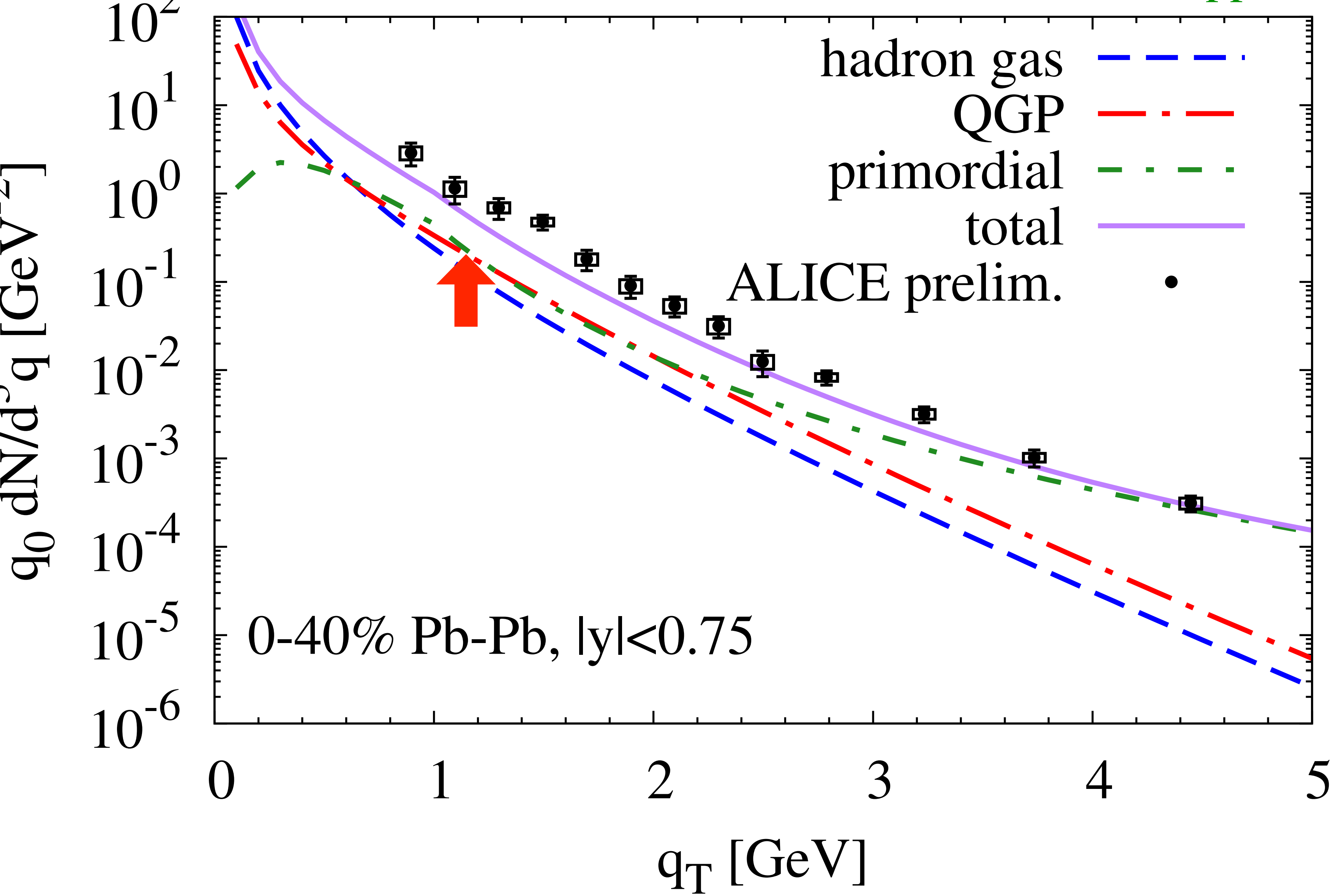


ALICE vs theory: puzzle of the "missing" photons

At LHC, "primordial" appears to dominate above $p_t \sim 1$ GeV

Again, experiment much larger than theory?

van Hees, He, Rapp, 1404.2846



Hadronic contribution to photons?

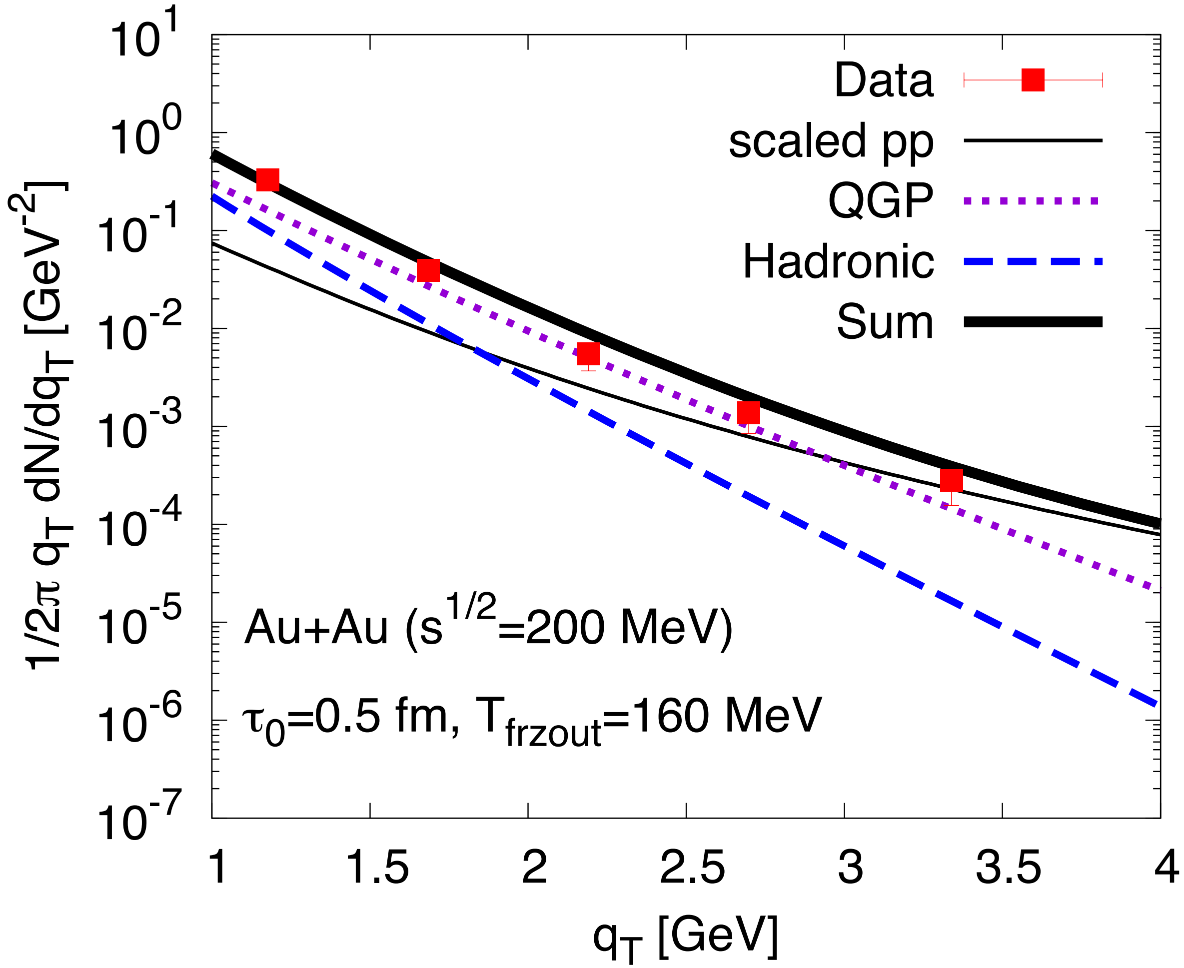
Dusling & Zahed, 0911.2426

Do virial expansion, need $\langle\pi|J_V(x)J_V(0)|\pi\rangle$; $\langle\pi\pi|J_V(x)J_V(0)|\pi\pi\rangle$

Use experimental input (R, τ decay) :

find hadronic contribution much larger than other analyses;

Resolves puzzle of the "missing" photons?



Shear viscosity: strongly suppressed

Shear viscosity in the semi-QGP

Shear viscosity, η, in the complete QGP:

Arnold, Moore & Yaffe, hep-ph/0010177 & 0302165 = AMY.

Generalize to $Q \neq 0$: Boltzmann equation in background field.

$$\eta = \frac{\mathcal{S}^2}{\mathcal{C}}$$

S = source, C = collision term. *Two* ways of getting small η:

"Strong" QGP, *large* coupling $S \sim 1$, $C \sim (\text{coupling})^2 >> 1$.

$\mathcal{N} = 4$ SU(N), $g^2 N = N = \infty$: $\eta/s = 1/4\pi$. Kovtun, Son & Starinets hep-th/0405231

Semi-QGP: small *loop* at *moderate* coupling: Pisarski & Hidaka, 0803.0453, 0912.0940

Pure glue: $S \sim \langle loop \rangle^2$, $C \sim g^4 \langle loop \rangle^2$

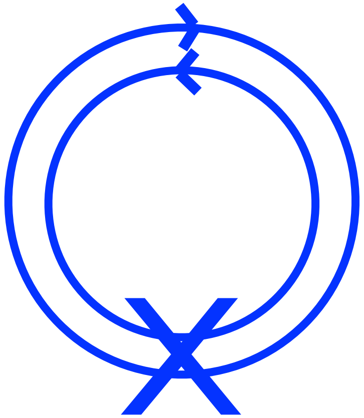
With quarks: $S \sim \langle loop \rangle$, $C \sim g^4$

Both: $\eta \sim \langle loop \rangle^2$

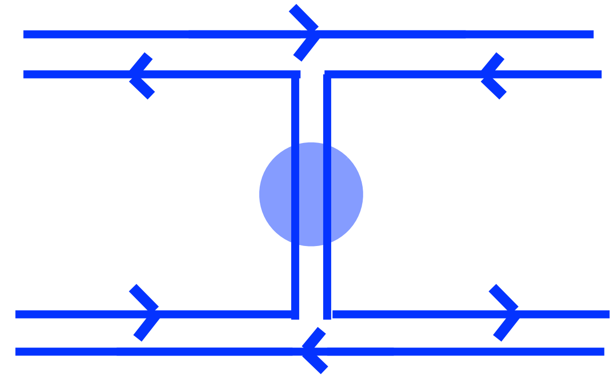
To leading log order: # from AMY, constant "c" beyond leading log

$$\frac{\eta}{T^3} = \frac{\#}{g^4 \log(c/g)} \mathcal{R}(\ell) \quad ; \quad \mathcal{R}(\ell \to 0) \sim \ell^2$$

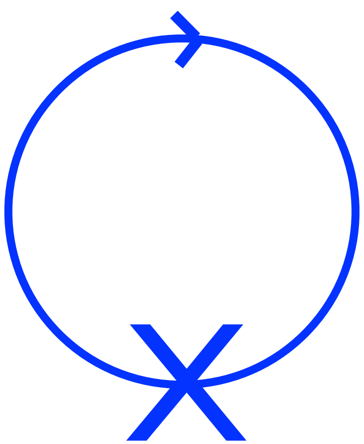
Counting powers of <*loop*> = *l* → 0



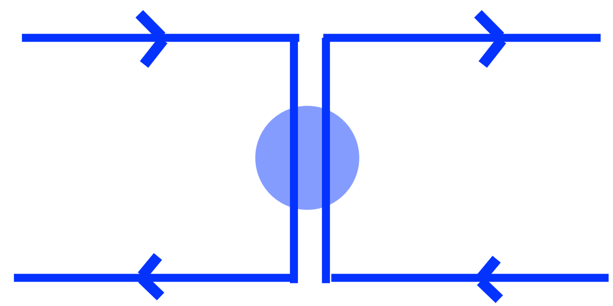
$\mathcal{S} \sim \ell^2$



$\mathcal{C} \sim \ell^2$

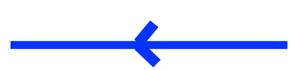


$\mathcal{S} \sim \ell$



$\mathcal{C} \sim 1$

$\sim e^{+iQ^a/T}$

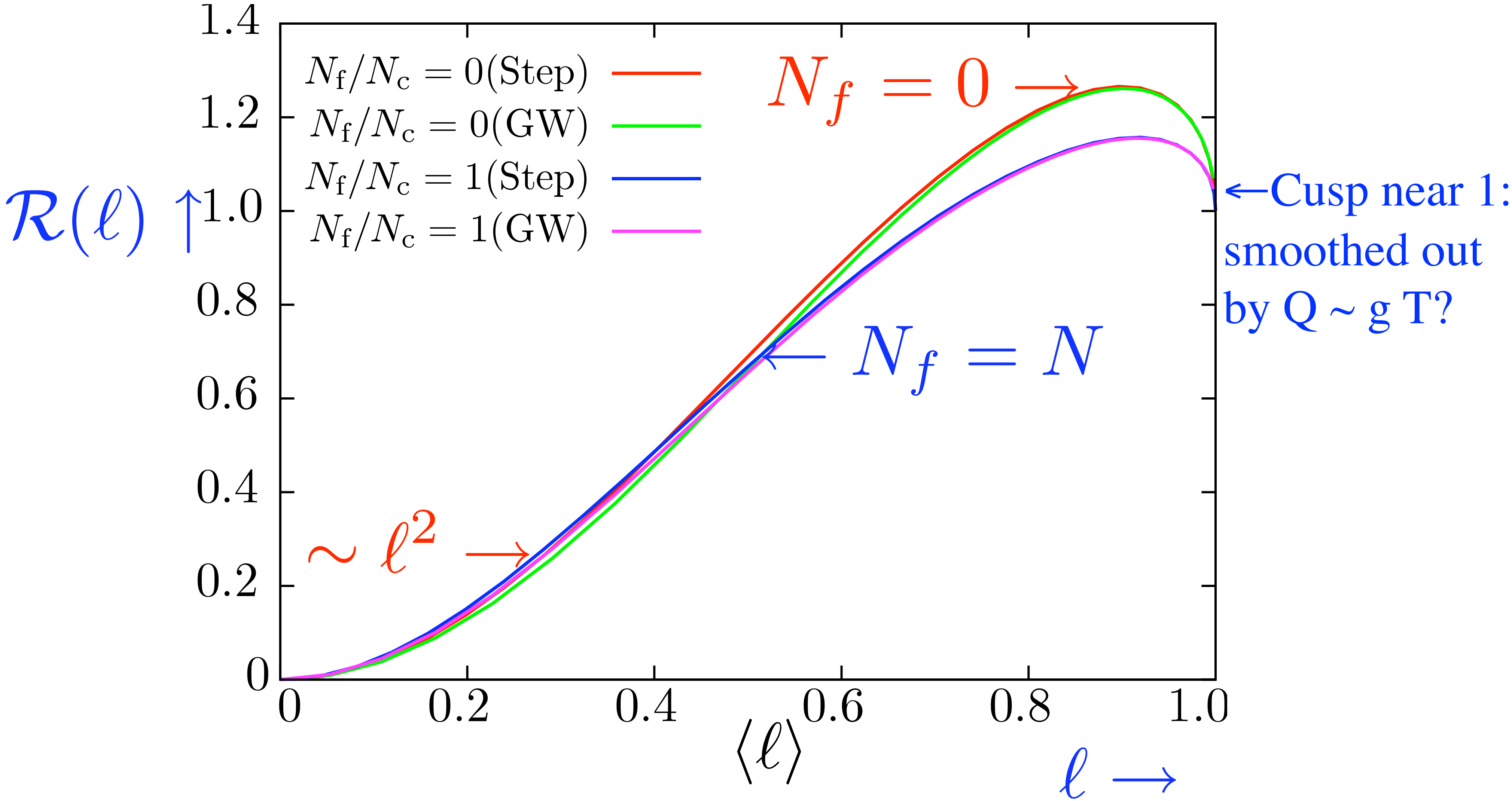


$\sim e^{-iQ^a/T}$

Small shear viscosity from color evaporation

R = ratio of shear viscosity in semi-QGP/complete-QGP at *same* g, T.
Two different eigenvalue distributions give *very* similar results!

When <*loop*> ~ 0.3, R ~ 0.3.



Shear viscosity/entropy

Leading log shear viscosity/lattice entropy. $\alpha_s(T_c) \sim 0.3$.

Large increase from T_c to 2 T_c. *Clearly* need results beyond leading log.

Also need to include: quarks and gluons *below* T_c, hadrons *above* T_c. *Not* easy.